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Examining students' geometrical misconceptions by eye tracking

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ABSTRACT

This study was conducted to reveal potential sources of students' difficulty and misconceptions about geometrical concepts with the help of eye tracking. In this study, the students' geometrical misconceptions were explored by answering the questions on the geometry test prepared based on the literature and test-taking processes and represented with the help of eye tracking data to make interpretations about their visual and cognitive attention. With this aim, the mixed method research design was used. The data were collected through the geometry test, eye tracking measurements, and the think aloud strategy. Geometry test scores were represented by descriptive statistics such as percentage. The quantitative eye tracking data were analysed by Kruskal–Wallis tests and Mann–Whitney *U*-tests among the choices for each question. The qualitative data collected through the think aloud strategy were analysed by the content analysis. In the findings, the students' geometrical misconceptions were examined and reported in detail from a different perspective with the help of data related to eye tracking. Then, eye tracking data and qualitative data showed the geometry misconception items on which the students had difficulty and cognitive conflict in addition to geometrical misconception based on the students' answers to the geometry questions.

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KEYWORDS
Cognitive conflict; eye tracking; geometrical misconception; students

1. Introduction

Because students are always in contact with geometry in daily life, geometry takes an important role in the school mathematics curriculum. French (2004) emphasizes the necessity of learning and teaching geometry by explaining its effects on students 'to extend spatial awareness, to develop the skills of reasoning, to stimulate, challenge and inform' (p. 2). Hence, students are expected to acquire and develop a conceptual understanding of the geometrical content by geometrical reasoning. Also, learning geometry necessitates a sense of representations of these contents by relating the shapes to their definitions (French, 2004; van de Walle, 2004). Because of various reasons, such as the need for geometrical intuition, relational concepts and consecutive, cumulative and abstract nature of mathematics, the

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students may learn the concepts inaccurately and misconceptions may arise (Ercan, 2010; French, 2004; Yenilmez & Yasa, 2008). Misconceptions can obstruct understanding of the concept and solving problems and cause learning later concepts inaccurately giving rise to other misconceptions (McDermott, 1991; Sandir, Ubuz & Argun, 2007; Sims-Knight & Kaput, 1983). Moreover, the studies show that the misconceptions can become permanent in students' conceptual development and would cause obstacles in their adulthood stage (Stacey et al., 2001). Unfortunately, students' formation of misconceptions has been one of the most common problems observed in the teaching and learning process (Kalchman, Moss & Case, 2001). At that point, it is essential to identify their misconceptions as soon as possible (Ayyildiz & Altun, 2013; Osborne & Gilbert, 1980; Resnick et al., 1989) since misconceptions can become more complex due to being in connection to and used in learning later relational concepts (Comins, 1998).

In the literature, there have been many research studies focusing on mathematical misconceptions because of their critical importance to teaching and learning (Ercan, 2010; McDermott, 1991; Sandir, Ubuz & Argun, 2007; Sims-Knight & Kaput, 1983). The identification and exploration of geometrical misconceptions have been examined by designing instructions and constructing tests based on the van Hiele theory (Luneta, 2015; Mbusi, 2019; Mbusi & Luneta, 2021; Žilková, Gunčaga & Kopáčová, 2015), written analysis (Kembitzky, 2009), problem-solving, engaging in multiple representations and virtual manipulatives (Hwanng et al., 2009), using technology (Mwangi, Githua, & Changeiywo, 2021) and engaging in tasks designed by the dynamic geometry software (Martinovic & Manizade, 2020). Some of the research studies explain the reasons for misconceptions as students' experiences in learning, perception and understanding of prior knowledge inaccurately or insufficiently, the discrepancy between the concept and its usage in real life, inappropriate teaching environments, opportunities or tasks, failure in relating concepts with other concepts or real life and inadequacy in identifying the students' misconceptions (Ayas & Demirbaş, 1997; Foster, 2007; Lawson & Thompson, 1988; Yenilmez & Yasa, 2008). Because the misconception is cognitively taking a concept into a place scientifically different from its accurate meaning rather than opposing wrong results or having insufficient knowledge, they may result from personal issues (Yenilmez & Yasa, 2008). Hence, further research may be necessitated to examine the reasons for these misconceptions focusing on these personal cognitive processes in a different way. Eye tracking can provide detailed information about this process and propose a different window to look into. Research has shown that eye tracking is related to cognitive structures since there is a connection between how individuals behave and what they think (Hyöna, 2010; Jang et al., 2014; Jarodzka, Scheiter, Gerjets, & Van Gog, 2010; Lowe & Boucheix, 2011; Mayer, 2010; Ozcelik, Karakus, Kursun, & Cagiltay, 2009; Patrick, Carter, & Wiebe, 2005; She & Chen, 2009; Just & Carpenter, 1980; Rayner, 1998). For example, the case that an individual fixing his eyes on a point can be interpreted as the process of intense thinking. Also, the fixation duration can be referred to as the duration necessitated to give the symbols and visuals meaning. Hence, it can be stated that eye tracking methods are useful to examine directly and acquire more detailed information about the underlying cognitive processes, and thoughts generated in the mind by providing attention-related evidence (Johnson & Mayer, 2012; Rayner, 1998).

In the previous research about the examination of students' geometrical misconceptions, the data were limited to the students' explanations, written documents, discussions and completed tasks. In other words, geometrical misconceptions were determined by

what they represent and show in the tasks or tests. However, there may be parts or factors on the task or test that they are not aware of or they did not explain. The students may not explain or not be aware of which parts they focus on or what distracts them on the task or test item. At this point, by obtaining these kinds of data through eye tracking, students' geometrical misconceptions can be examined in detail and clearly. In the light of the given explanations earlier, it was aimed to extend the research of examining geometrical misconceptions with the help of eye tracking. The current study differentiates from the previous research about geometrical misconceptions by using eye tracking. Hence, the current study was conducted to explore students' geometrical misconceptions, and visual and cognitive attention to solving multiple choice and matching geometry problems by the eye tracking technology. More specifically, the students solved these geometry problems prepared based on the commonly observed geometrical misconceptions stated in the literature. In this solving process, the spending their time, the reasons for their selection of the items and factors affecting this selection were examined by comparing the visual and cognitive attention of the students with or without misconceptions. Hence, the research problems of the current study were as follows:

(1) Is there a statistically significant difference in the amount of attention that the student pay to the inspecting choices in geometry questions?
(2) How do the students spend time the inspecting choices in geometry questions?
(3) What are the factors affecting the students' selection of the particular inspecting choices in geometry questions?

2. Theoretical framework

2.1. Geometrical misconceptions

Misconceptions are an evitable and natural part of learning and conceptual development. Hansen (2017) emphasizes this issue by stating that 'as children develop their own understanding of the world they will unavoidably misconceive some ideas' (p. 2), and the reason for making mistakes is defined as 'natural stage of conceptual development' by Swan (2001, p. 154). Previous research has shown that students have many misconceptions affecting their understanding and learning of mathematical concepts and preventing construction of new conceptual learning (Egodawatte, 2011, p. 261). More specifically and narrowly, Luneta (2015) emphasizes that 'the majority of students did not understand most of the basic concepts' in Euclidian geometry. The main important and difficult tasks are identifying the underlying reasons for misconceptions (Nesher, 1987) because it becomes possible to design and organize the instructions and tolls to overcome them successfully. The factors of difficulty in identifying misconceptions, their nature of being resilient and students' resistance to holding them limit to determine the reasons behind the formation of misconceptions (Xiaobao, 2006). When the literature about the geometrical misconceptions has been examined, the misconceptions about the concepts of angle and angle measure (Devichi ve Munier, 2013; Doyuran, 2014; Ozen-Unal & Urun, 2021), the concept and definition of a polygon (Akuysal, 2007), a triangle (Haj-Yahya, 2021; Tsamir, Tirosh & Levenson, 2008), quadrilaterals and their hierarchical relationship (Monaghan, 2000;

Pickreign, 2007; Erez & Yerushalmy, 2006; Fujita & Jones, 2008; Birgin & Özkan, 2022) and a prism (Hasanah & Yulianti, 2020; Tsamir et al., 2014) have come forward.

2.2. Eye tracking studies in mathematics education

Eye tracking proposes various opportunities for mathematics education research (Andrá et al, 2015; Obersteiner & Tumpek, 2016). In the literature, there are many research studies related to mental operations with the help of eye tracking to gain insight into the cognitive process taking place in completing the tasks and tests (Andrá et al, 2015; Jacob & Karn, 2003; Obersteiner & Tumpek, 2016; Rayner, 1998). For example, individuals' mathematical problem-solving processes and going through problem-solving steps were explored by eye tracking (Ellis et al., 2011; Knoblich, Ohlsson & Raney, 2001; Thomas & Lleras, 2007). Moreover, eye tracking provides to gain insight into metacognition (Cohors-Fresenborg et al., 2010). Also, creativity in solving geometrical problems (Muldner & Burleston, 2015; Schindler et al., 2016) is explored by mostly paying attention to 'how and which information are attending to' (Andra et al., 2015, p. 241). Hence, it becomes possible to analyse visual attention on completing tasks even if they necessitate simple motor actions such as clicking on an item using a mouse (Susac, Bubic, Kaponja, Planinic & Palmovic, 2014). Furthermore, the previous research shows that patterns of eye tracking can provide information about relational thinking processes referring to relational schemata (Thomas & Lleras, 2007). Therefore, eye tracking provides to gain insight into the students' reasoning, cognitive conflict and difficulty with multiple-choice items (Schindler et al., 2016; Muldner & Burleston, 2015; Tsai, Hou, Lai, Liu & Yang, 2012). Cognitive conflict refers to the conflict or mismatch occurring between new knowledge and existing information through new experiences or the scientific models to be learned from the text (Mikkilä-Erdmann, Penttinen, Anto & Olkinuora, 2008). At this point, old or existing conceptions are not suitable or sufficient while engaging in these new experiences (Posner et al., 1982).

Given these explanations show that eye tracking can enhance giving meaning and acquiring insight into the cognitive processes including cognitive conflict through solving problems and answering questions in geometry. Hence, in this study, it was aimed to focus on how to interpret eye tracking data acquired about the mathematical sub-domain of geometry. Moreover, the students' answering question processes were explored using eye tracking paying attention to geometrical misconceptions in this study so that it was aimed to extend previous research about solving geometry problems by focusing on geometrical misconceptions and eye tracking measurements.

2.3. Interpreting eye tracking: evidence for geometry education research

The eye tracking technology performs some measurements such as fixations (i.e. moments that eyes remain focused on an area). Fixations as the most commonly used measurement can provide information about cognitive attention and difficulty in taking up information and commentating (Andra et al., 2015; Jacob & Karn, 2003). Previous research has shown that an increase in fixation time and numbers is related to the occurrence of cognitive difficulty in completing tasks and tests (Epelboim & Suppes, 2001; Liversedge & Findlay, 2000). Conversely, it can be stated that the more expertise the individuals attain in particular subject areas, the fewer eye fixations are observed. It is emphasized that 'what a

person looks at is assumed to indicate the thought "on top of the stack" of cognitive processes (Jang et al., 2014, p. 318). Because of the differences among individuals' cognitive structures, there are differences among eye tracking measurements both within and across individuals (Andrews & Coppola, 1999; Castelhano, Mack, & Henderson, 2009; Henderson & Hollingworth, 1999; Rayner, 1998). Galley, Betz and Biniossek (2015) illustrate this variability by separating the fixation duration into four categories: very short fixations (shorter than 90 ms referring to the little cognitive process), express fixations (90–150 ms referring to simple cognitive process including looking at visuals), cognitive fixations (150–900 ms referring to cognitive operations) and rare very long fixations (more than 900 ms referring to being not in cognitive process). Hence, longer fixations can be interpreted as being in the effortful cognitive process, engaging in the complicated task including complex items, the importance of the tasks and having difficulty in extracting information (Goldberg & Kotval, 1999; Harris & Christhilf, 1980; Henderson et al., 1999; Rayner, 1998; Rayner & Pollastek, 1989). All of these measure-related cognitive processes can provide information about cognitive conflicts if they exist. Moreover, eye tracking can provide information about the cognitive process in answering multiple-choice questions by comparing the actions of successful and unsuccessful test takers. Previous research shows that successful test takers tend to spend more time on relevant choices than other irrelevant choices in answering multiple-choice questions (Tsai et al., 2012). Moreover, Lin and Lin (2014) have shown that some eye tracking measures such as well time, fixation count, and run count change for successful and unsuccessful problem solvers. They have found that these measures are correlated with the students' perceived difficulty. Furthermore, the eye tracking measures of dwell time and fixation count represent how much time and which parts the solver's attention is paid on. The longer dwell time or higher fixation count the solver has, the more cognitive capacity he/she requires (Carpenter & Just, 1978). In another study, Hegarty, Mayer and Green (1992) found that the students providing accurate solutions to the problems had lower scores on eye tracking measurements related to fixation than the ones providing inaccurate solutions. Hence, the interpretation of these correlations provides detailed information about the students' difficulties with diagrams, problems and figures. Based on these explanations, differences among eye tracking data collected through the students having geometrical misconceptions, their difficulties and not having misconceptions can be observed.

3. Method

To answer research questions and propose detailed information about the students' geometrical misconceptions, the study was designed by the sequential explanatory mixed method research design. In this design, the quantitative data were collected to examine the misconceptions, and then qualitative data were gathered to look for a detailed and holistic picture and the reasons for these misconceptions. Hence, the study was composed of two parts: qualitative and quantitative.

3.1. Participants

In the quantitative part of the study, the multiple choice and matching geometry test were conducted on 386 seventh grade students. These participants were selected by the typical

sampling strategy. In this respect, different types of schools from different districts of a cosmopolite province were selected. Five schools from low, middle and high socioeconomic level districts were selected. Then, all of the seventh grade students performed the test. Approximately 55% of 386 students were females and 45% of them were males. Afterwards, in the qualitative part of the study, 30 students were selected using a cluster sampling strategy by identifying 6 students from each school. In each school, three male and three female students were selected. These students did not take a role in the quantitative part of the study. They answered the questions on the computer screen by the think aloud strategy.

3.2. Materials

The test was prepared by a group of researchers having Ph.D. degrees in mathematics education and teaching technology. The questions of this test were represented to the students using web 2.0 tools on a computer screen. On the screen, eight multiple-choice questions and one matching question (see Appendix for sample questions) were represented to the students. They selected the answers by distinguishing relevant and irrelevant items and then clicked to switch to the next question. They answered questions one by one. The questions were prepared by examining the mathematics middle school curriculum, geometry textbooks and studies about geometrical misconceptions in the literature. Based on the literature, the common and mostly observed geometrical subjects and contexts in which the students misconceptions and difficulties were determined. In the mathematics middle school curriculum, the number of objectives for each geometrical concept was determined. Then, the literature about the geometrical misconceptions was examined, and the commonly observed geometrical misconceptions were determined. The questions of the current study were selected from the questions used in the previous research more than once. The questions were designed about angle and angle measure (Devichi ve Munier, 2013; Doyuran, 2014), the concept and definition of polygon (Akuysal, 2007), a triangle (Clements et al., 1999; Tsamir, Tirosh & Levenson, 2008; Tsamir, Tirosh, Levenson, Barkai & Tabach, 2014), a triangle and altitude of triangles (Alatorre & Saiz, 2009; Uygun & Akyuz, 2019; Hershkowitz, 1990), quadrilaterals and their hierarchical relationship (Monaghan, 2000; Pickreign, 2007; Erez & Yerushalmy, 2006; Fujita & Jones, 2008) and a prism and opening form of geometric solids (Gökkurt, Şahin, Soylu & Doğan, 2015; Tsamir et al., 2014). Through answering these questions, the process of eye tracking was recorded by SMI Experiment 2 to represent visual stimuli.

3.3. Data collection and data analysis

The data collection process lasting 2 months was performed in two parts, and quantitative and qualitative data were focused on the current study. Initially, the students solved multiple-choice and matching geometry problems performed by the researchers on the website. Quantitative data were collected through all of the participants' answering questions and eye gaze data. The answers of the students and eye tracking data belonging to these answering processes were recorded by the eye tracking tool. The quantitative data collection process for each student lasted for approximately 30 min. This process was performed as illustrated in Figure 1. The students' eye tracking data, selection of answers and

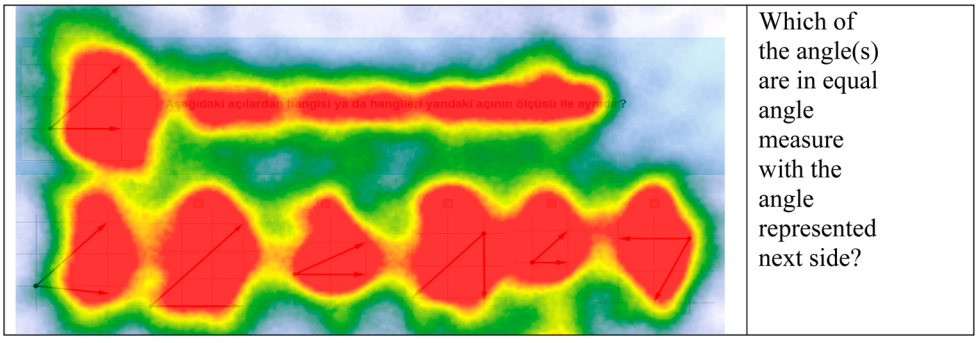


Figure 1. Heat map for the question about the angle.

actions taken by the computer such as mouse clicking and duration of answering questions were all analysed and reported by the BeGaze program. Also, their answering processes were examined using eye tracking strategies considering their attention by separating the computer screen into specific areas [i.e. areas of interest (AOI)] and the sequence patterns of their fixation. These AOIs were determined based on the number of alternative choices on each item and the question. Totally, 65 AOIs were identified for data analysis of the study. These AOIs were labelled as the choices proposed on the questions. In the current study, non-parametric tests were conducted to analyse eye tracking data because these data do not distribute normally. The case of non-normally distributed eye tracking data has been observed in previous research (Lin & Lin, 2014, 2018). For each question, Kruskal–Wallis analysis was performed using the eye tracking data [including visible time (m/s), dwell time (m/s), glance duration (m/s), diversion duration (m/s), glances count, revisits, fixation count, fixation time (m/s)] based on the items for each question. Also, Mann–Whitney *U*-tests were performed among the choices of each question.

In the qualitative data collection process, 30 students were asked to answer the questions on the computer screen by the think aloud strategy. Before taking the test, the students were explained how to answer the questions by thinking aloud and how to provide justification for answer selection. They explained 'Why and how did they select the particular choices?, How and why did they eliminate the other choices?'. Hence, the data about the justification for the students' answering could be acquired. The qualitative data collection process for each student lasted approximately for 45 min. The students' expressions made through think aloud strategy were recorded by audio recordings during the whole answering process. Then, the data were transcribed verbatim. To analyse these data on these transcripts, the content analysis technique was performed by the researchers. The researchers analysed the transcripts independently to form the list of codes and themes. Then, they came together to form a common list of codes and themes. They accepted the common codes and themes, and they discussed the differences among them until they reached a consensus. Hence, the list of codes and themes were identified with 90% consistency among the researchers. The remaining parts were eliminated since they were not accepted as necessary to take place in the study. Also, by investigator and data triangulation technique, the evidence for reliability and validity of the current study was provided.

Table 1. Kruskal–Wallis results based on AOI measures for the first question.

AOI measure type	*N*	df	X^2	*p*	Significant difference with higher measure
Visible time	1672	5	1.486	.000	Question, second and fourth choices for other choices
Dwell time	1672	5	450.919	.000	Question, second and fourth choices for other choices
Glance duration	1672	5	478.543	.000	Question, second and fourth choices for other choices
Diversion duration	1672	5	491.830	.000	Question, second and fourth choices for other choices
Glances count	1672	5	335.187	.000	Question, second and fourth choices for other choices
Revisits	1672	5	335.187	.000	Question, second and fourth choices for other choices
Fixation count	1672	5	573.654	.000	Question, second and fourth choices for other choices
Fixation time	1672	5	424.401	.000	Question, second and fourth choices for other choices
AOIs	1672	5	0.915	.000	Question, second and fourth choices for other choices

4. Findings

To examine the differences among eye tracking measures of the students based on the choices of the questions, AOIs were produced. The number of choices and the question were the determinants of the number of AOIs for each question. In other words, of the number of AOIs in each question, one of them was the question statement and the others were the choices to be selected. The measures of visible time (m/s), dwell time (m/s), glance duration (m/s), diversion duration (m/s), glances count, revisits, fixation count and fixation time (m/s) were recorded. For each question, the question part and the choices that the students were expected to analyse and identify as the correct answer were compared based on the measurement acquired through eye tracking measurements using Kruskal–Wallis and Mann–Whitney *U*-test analysis. Then, the statistically significantly different part was identified as the most difficult party for the students. In other words, based on the eye tracking measurements analysed by Kruskal–Wallis and Mann–Whitney *U*-test analysis, the differences based on the choices were examined in line with the cases of whether the students easily understand the question/identify the correct answer or on which parts they had difficulty. Moreover, the heat maps for each question were provided to illustrate the eye tracking data based on the choices of the questions more clearly. In this section, the findings about most of the questions on the test were illustrated (see the link presented in Appendix – Geometry test for the questions and the options for each question).

In the question prepared about the concepts of angle and angle measures, there are six inspecting choices. Of the students, 51% selected the second choice, 21% clicked the fifth option and 61% chose the fourth choice as the true answers accurately. Seven AOIs were identified in this question. For this question, Kruskal–Wallis tests performed on these scores revealed statistically significant differences in eye tracking measures as in Table 1.

Of the students across seven AOIs ($n = 1672$) $= .915$, X^2 (5, $N = 1672$) $= 1.486$ for visible time (m/s), X^2 (5, $N = 1672$) $= 450.919$ for dwell time (m/s), X^2 (5, $N = 1672$) $= 478.543$ for glance duration (m/s), X^2 (5, $N = 1672$) $= 491.830$ for diversion duration (m/s), X^2 (5, $N = 1672$) $= 335.187$ for glances count, X^2 (5, $N = 1672$) $= 335.187$ for revisits, X^2 (5, $N = 1672$) $= 573.654$ for fixation count and X^2 (5, $N = 1672$) $= 424.401$ for fixation time (m/s) with $p \leq .01$. According to Mann–Whitney *U*-tests, the AOI referring to question, the second and fourth choices got higher median score for dwell time recorded consecutively as a higher median score of ($Md = 1304.91$), ($Md = 852.24$) and ($Md = 883.04$); the question with ($Md = 1317.97$), the second choice with ($Md = 853.81$) and the fourth choice with ($Md = 892.57$) for

Table 2. Kruskal–Wallis results based on AOI measures for the second question.

AOI measure type	*N*	df	X^2	*p*	Significant difference with higher measure
Visible time	2456	7	751.22	.000	Third choice for other choices
Dwell time	2456	7	214.354	.000	Third choice for other choices
Glance duration	2456	7	210.236	.000	Third choice for other choices
Diversion duration	2456	7	208.621	.000	Third choice for other choices
Glances count	2456	7	293.936	.000	Third choice for other choices
Revisits	2456	7	293.936	.000	Third choice for other choices
Fixation count	2456	7	167.192	.000	Third choice for other choices
Fixation time	2456	7	216.784	.000	Third choice for other choices

glance duration; the question with ($Md = 1326.64$), the second choice with ($Md = 855.01$) and the fourth choice with ($Md = 889.52$) for diversion duration; the question with ($Md = 1185.50$), the second choice with ($Md = 913.03$) and the fourth choice with ($Md = 963.83$) for glance count; the question with ($Md = 1185.50$), the second choice with ($Md = 913.03$) and the fourth choice with ($Md = 963.83$) for revisits; the question with ($Md = 1369.01$), the second choice with ($Md = 881.78$) and the fourth choice with ($Md = 894.90$) for fixation count; the question with ($Md = 1287.50$), the second choice with ($Md = 853.46$) and the fourth choice with ($Md = 885.36$) for fixation time (m/s) than other AOIs.

Through think aloud strategy, a student-made an explanation for her answer as follows:

R: Why do you select these choices? (Showing the second, fourth and the fifth options)

S_1: Because the figure includes four squares as in it is represented and it is extended. The fourth option looks like the figure, it is rotated, I think. Iımm, I am not sure about the fifth one, but it may not be true … However, in the figure, there are two squres vertically and two squares horizontally. Similarly but in smaller case, on the fifth figure, one horizontal square and one vertical square. Then, it might be true.

R: Are you sure?

S_1: Ummmm … It might be true … Yes, it is true. I am sure.

Of the students, 68% selected the first choice, 60% selected the third choice and 92% selected the sixth choice truly. For the question prepared about the concept of the triangles, eight AOIs were identified. The results of Kruskal–Wallis test are presented in Table 2.

Kruskal–Wallis tests, on the scores, revealed statistically significant differences in eye tracking measures of the students across eight AOIs ($n = 2456$), $p = .000$, X^2 (7, $N = 2456$) = 751.22 for visible time (m/s), X^2 (7, $N = 2456$) = 214.354 for dwell time (m/s), X^2 (7, $N = 2456$) = 210.236 for glance duration (m/s), X^2 (7, $N = 2456$) = 208.621 for diversion duration (m/s), X^2 (7, $N = 2456$) = 293.936 for glances count, X^2 (7, $N = 2456$) = 293.936 for revisits, X^2 (7, $N = 2456$) = 167.192 for fixation count and X^2 (7, $N = 2456$) = 216.784 for fixation time with $p = .000$. According to Mann–Whitney *U*-tests, the AOI referring to the third choice got higher median score for visible time recorded as a higher median score of ($Md = 1453.75$), the third choice with ($Md = 1454.33$) for dwell time, the third choice with ($Md = 1461.02$) for

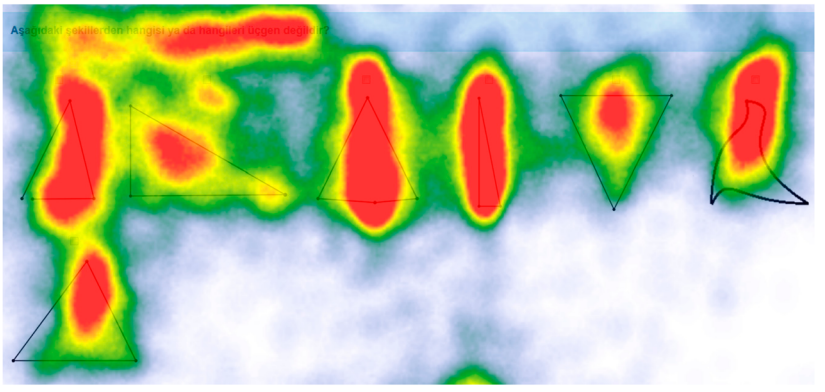


Figure 2. Heat map for the question about triangles.

Table 3. Kruskal–Wallis results based on AOI measures for the third question.

AOI measure type	*N*	df	X^2	*p*	Significant difference with higher measure
Visible time	2221	7	1.526	.000	First and third choices for other choices
Dwell time	2221	7	165.041	.000	First and third choices for other choices
Glance duration	2221	7	173.515	.000	First and third choices for other choices
Diversion duration	2221	7	180.699	.000	First and third choices for other choices
Glances count	2221	7	145.572	.000	First and third choices for other choices
Revisits	2221	7	145.572	.000	First and third choices for other choices
Fixation count	2221	7	218.111	.000	First and third choices for other choices
Fixation time	2221	7	156.255	.000	First and third choices for other choices

glance duration, the third choice with ($Md = 1463.55$) for diversion duration, the third choice with ($Md = 1557.59$) for glance count, the third choice with ($Md = 1557.59$) for revisits, the third choice with ($Md = 1435.14$) for fixation count and the third choice with ($Md = 1459.59$) for fixation time than other AOIs. In the think aloud process, a student-made explanation is given as follows:

> S_5: The sixth option does not completely look like a triangle. In the first choice, it looks like a triangle but it is not a triangle since the line segments must be connected. The third option looks like a triangle. Ummm … but it is not a triangle since the lşne segment on the base must be linear. It is not linear, it is broken. Uppss.. there are two line segments there.

Although most of the students identified the fourth choice accurately, they had difficulty in determining it as a triangle as observed in Figure 2, representing the heat map of the answering the question. A student explained that 'it is very tight, I am not sure … but even if it is very tight, it is a triangle since it has three vertices and three edges'.

The first option was selected by 54% of the students, the third choice by 64% and the sixth choice by 19%. For the question prepared about the concept of altitudes of triangles, Kruskal–Wallis tests performed on these scores revealed statistically significant differences in eye tracking measures of the students, as represented in Table 3.

Across eight AOIs ($n = 2211$), X^2 $(7, N = 2211) = 1.526$ for visible time (m/s), X^2 $(7, N = 2211) = 165.041$ for dwell time (m/s), X^2 $(7, N = 2211) = 173.515$ for

glance duration (m/s), X^2 (7, $N = 2211$) = 180.699 for diversion duration (m/s), X^2 (7, $N = 2211$) = 145.572 for glances count, X^2 (7, $N = 2211$) = 145.572 for revisits, X^2 (7, $N = 2211$) = 218.111 for fixation count and X^2 (7, $N = 2211$) = 156.255 for fixation time (m/s) with $p = .000$. According to Mann–Whitney U-tests, the AOI referring to the first and third choices got a higher median score for dwell time recorded as a higher median score of ($Md = 1369.35$, $Md = 1341.93$), the first choice with ($Md = 1377.18$) and third choice with ($Md = 1351.37$) for glance duration, the first choice with ($Md = 1384.02$) and third choice with ($Md = 1355.10$) for diversion duration, the first choice with ($Md = 1233.59$) and the third choice with ($Md = 1366.58$) for glance count, the first choice with ($Md = 1233.59$) and the third choice with ($Md = 1366.58$) for revisits, the first choice with ($Md = 1372.79$) and the third choice with ($Md = 1380.67$) for fixation count and the first choice with ($Md = 1363.18$) and the third choice with ($Md = 1339.94$) for fixation time than other AOIs.

Through the think aloud process, a student selecting the first and third choices accurately and not selecting the sixth choice inaccurately made the following explanation:

S_{11}: The first choice looks like accurate… (after thinking for a while) but I want to change my answer.
R: Why?
S_{11}: In the first option, the line segment referring to h is extended more than needed, it should not exceed the extension of the edge of BC… .
R: What about the others?
S_{11}: … on the third option, h does not exceed the edge and it intersects the edge. I have not seen a figure like this. It may not be true.
R: How can you draw the altitude truly for this triangle?
S_{11}: I need to draw a line segment from the vertex C to the edge of AB.
R: … is it happened on it?
S_{11}: I need to form right angle but there is not.
R: Is there any other option that you select?
S_{11}: No.
R: What about the sixth one?
S_{11}: It is true. It looks like accurate figure since a line segment from vertex to the edge and there is right angle.

Twenty-two percent of the students placed parallelogram, 20% did square, 17% did trapezoid, 11% did rectangle and 19% rhombus accurately. Most of the students had difficulty in explaining their answers even if they answered accurate parts. For the question prepared about the hierarchical relationship among quadrilaterals, Kruskal–Wallis tests performed on these scores revealed statistically significant differences in eye tracking measures of the students across six AOIs ($n = 175$) as represented in Table 4.

Based on the measures, X^2 (6, $N = 175$) = 1.113 for visible time (m/s), X^2 (6, $N = 175$) = 40.606 for dwell time (m/s), X^2 (6, $N = 175$) = 57.184 for glance duration (m/s), X^2 (6, $N = 175$) = 62.113 for diversion duration (m/s), X^2 (6, $N = 175$) = 47.843 for glances count, X^2 (6, $N = 175$) = 47.843 for revisits, X^2 (6, $N = 175$) = 52.535 for fixation count and X^2 (6, $N = 175$) = 40.437 for fixation time (m/s) with $p = .000$. According to Mann–Whitney U-tests, the AOI referring to the figure got higher median

Table 4. Kruskal–Wallis results based on AOI measures for the fourth question.

AOI measure type	*N*	df	X^2	*p*	Significant difference with higher measure
Visible time	175	6	1.113	.000	The figure for other choices
Dwell time	175	6	40.606	.000	The figure for other choices
Glance duration	175	6	57.184	.000	The figure for other choices
Diversion duration	175	6	62.113	.000	The figure for other choices
Glances count	175	6	47.843	.000	The figure for other choices
Revisits	175	6	47.843	.000	The figure for other choices
Fixation count	175	6	52.535	.000	The figure for other choices
Fixation time	175	6	40.437	.000	The figure for other choices

Note: The figure refers to the Venn diagram belonging to the question.

Table 5. Kruskal–Wallis results based on AOI measures for the fifth question.

AOI measure type	*N*	df	X^2	*p*	Significant difference with higher measure
Visible time	2129	7	0.782	.000	The second choice
Dwell time	2129	7	124.966	.000	The second choice
Glance duration	2129	7	126.449	.000	The second choice
Diversion duration	2129	7	122.648	.000	The second choice
Glances count	2129	7	193.272	.000	The second choice
Revisits	2129	7	193.272	.000	The second choice
Fixation count	2129	7	134.639	.000	The question, first, second and third choices
Fixation time	2129	7	120.410	.000	The question, first and second choices

score for dwell time recorded as a higher median score of ($Md = 131{,}92$), the figure with ($Md = 133.68$) for glance duration, the figure with ($Md = 134.04$) for diversion duration, the figure with ($Md = 128.36$) for glance count, the figure with ($Md = 128.36$) for revisits, the figure with ($Md = 135.64$) for fixation count and the figure with ($Md = 131.84$) for fixation time than other AOIs.

In the question prepared about the concepts of parallelograms, there are seven inspecting choices. Of the total students, 44% of the students selected accurately the first and the fifth options as not parallelograms. On the other hand, 40% of the students selected the second option and the fourth option and 41% chose the third option inaccurately. A student selecting the second option inaccurately stated that 'it is a quadrilateral because of its appearance and quadrilaterals are not parallelograms'. Another student making the same mistake explained that 'it is an example for rhombus, so it is not a parallelogram'. This student also selected the third and the fourth options inaccurately and stated that 'the third one is not a parallelogram since it is a square' and added that 'the fourth one is not either since it is a rectangle'. For this question, Kruskal–Wallis tests performed on these scores revealed statistically significant differences in eye tracking measures as presented in Table 5.

Based on the measures, X^2 (7, $N = 2129$) = 0.782 for visible time (m/s), X^2 (7, $N = 2129$) = 124.966 for dwell time (m/s), X^2 (7, $N = 2129$) = 126.449 for glance duration (m/s), X^2 (7, $N = 2129$) = 122.648 for diversion duration (m/s), X^2 (7, $N = 2129$) = 193.272 for glances count, X^2 (7, $N = 2129$) = 193.272 for revisits, X^2 (7, $N = 134.639$) = 134.639 for fixation count and X^2 (7, $N = 2129$) = 120.410 for fixation time (m/s) with $p = .000$. According to Mann–Whitney *U*-tests, the AOI referring to the second choice got a higher median score for dwell time recorded as a higher median

Table 6. Kruskal–Wallis results based on AOI measures for the sixth question.

AOI measure type	*N*	df	X^2	*p*	Significant difference with higher measure
Visible time	1655	5	0.869	.000	The second and fourth choices to other choices
Dwell time	1655	5	49.652	.000	The second and fourth choices to other choices
Glance duration	1655	5	57.184	.000	The second and fourth choices to other choices
Diversion duration	1655	5	62.113	.000	The second and fourth choices to other choices
Glances count	1655	5	169.548	.000	The second and fourth choices to other choices
Revisits	1655	5	169.548	.000	The second and fourth choices to other choices
Fixation count	1655	5	91.338	.000	The second and fourth choices to other choices
Fixation time	1655	5	48.126	.000	The second and fourth choices to other choices

score of ($Md = 1278.39$), the second choice with ($Md = 1278.57$) for glance duration, the second choice with ($Md = 1276.70$) for diversion duration, the second choice with ($Md = 1283.67$) for glance count and the second choice with ($Md = 1283.67$) for revisits than other AOIs. For the scores of fixation time, the question with ($Md = 1163.42$), the first choice with ($Md = 1184.25$) and the second choice with ($Md = 1282.01$) got higher median scores than other AOIs. For the scores of fixation count, the question with ($Md = 1207.28$), the first choice with ($Md = 1180.15$), the second choice with ($Md = 1190.83$) and the third choice with ($Md = 1159.73$) got higher median scores than other AOIs.

Sixty-four percent of the students selected the fourth choice and 26% did the second choice accurately. Forty-two percent of them selected the first choice, 16% did the third and 44% did the last choice inaccurately. The students selecting the first and third choices inaccurately explained '… since I could not imagine the other view that it is not seen. Hence, I feel that it could not be another triangle'. A student selecting the last choice inaccurately emphasized the oblique edges of the figure. Another student identifying the second choice as a prism inaccurately emphasized that 'the second one is cylinder so it must be prism'. For the question prepared about the concept of prisms, Kruskal–Wallis tests performed on these scores revealed statistically significant differences in eye tracking measures of the students across six AOIs ($n = 1655$) as represented in Table 6.

Based on the measurements, X^2 (5, $N = 1655$) = 0.869 for visible time (m/s), X^2 (5, $N = 1655$) = 49.652 for dwell time (m/s), X^2 (5, $N = 1655$) = 57.184 for glance duration (m/s), X^2 (5, $N = 1655$) = 62.113 for diversion duration (m/s), X^2 (5, $N = 1655$) = 169.548 for glances count, X^2 (5, $N = 1655$) = 169.548 for revisits, X^2 (5, $N = 1655$) = 91.338 for fixation count and X^2 (5, $N = 1655$) = 48.126 for fixation time (m/s) with $p = .000$. According to Mann–Whitney *U*-tests, the AOI referring to the second choice and the fourth choice got a higher median score for dwell time recorded as a higher median score of ($Md = 958.88$, $Md = 840.15$), the second choice with ($Md = 967.37$) and the fourth choice with ($Md = 832.83$) for glance duration, the second choice with ($Md = 970.43$) and the fourth choice with ($Md = 833.62$) for diversion duration, the second choice with ($Md = 978.68$) and the fourth choice with ($Md = 734.48$) for glance count, the second choice with ($Md = 978.68$) and the fourth choice with ($Md = 1024.88$) for revisits, the second choice with ($Md = 1030.44$) and the fourth choice with ($Md = 912.49$) for fixation count and the second choice with ($Md = 948.10$) and the fourth choice with ($Md = 906.04$) for fixation time than other AOIs.

Table 7. Kruskal–Wallis results based on AOIs measures for the seventh question.

AOI measure type	*N*	df	X^2	*p*	Significant difference with higher measure
Visible time	1654	5	2,537	.000	The second choice to other choices
Dwell time	1654	5	86.813	.000	The second choice to other choices
Glance duration	1654	5	95.307	.000	The second choice to other choices
Diversion duration	1654	5	104.351	.000	The second choice to other choices
Glances count	1654	5	114.457	.000	The second choice to other choices
Revisits	1654	5	114.457	.000	The second choice to other choices
Fixation count	1654	5	208.715	.000	The second choices to other choices
Fixation time	1654	5	81.036	.000	The second choice to other choices

Fifty percent of the students selected the first option, 53% did the second and 40% did the third option accurately. On the other hand, 22% of the students selected the fourth and 50% did the last option inaccurately. A student selecting the fourth option stated that '… if it is closed, a cube is formed since the opened form of a cube is drawn like this. This looks like the examples I have seen'. Another student selecting the last option explained that 'this one is true because it is a cylinder. I remember that it is a kind of prism'. A student explained the last figure as a round prism. When the researcher asked the reason for this answer, this student added that 'the appearance of the figure from the front and back views are the same'. Although one of the students selecting the second option explained, 'the bases have six sides, there are six rectangles, so it must be closed, and it becomes a prism' accurately, and none could decide about identifying it as a closed prism. They had difficulty in imagining its closed form. For the question prepared about this question, Kruskal–Wallis tests performed on these scores revealed statistically significant differences in eye tracking measures of the students across six AOIs ($n = 1654$) as presented in Table 7.

Based on the measurements, X^2 (5, $N = 1654$) = 2537 for visible time (m/s), X^2 (5, $N = 1654$) = 86.813 for dwell time (m/s), X^2 (5, $N = 1654$) = 95.307 for glance duration (m/s), X^2 (5, $N = 1654$) = 104.351 for diversion duration (m/s), X^2 (5, $N = 1654$) = 114.457 for glances count, X^2 (5, $N = 1654$) = 114.457 for revisits, X^2 (5, $N = 1654$) = 208.715 for fixation count and X^2 (5, $N = 1654$) = 81.036 for fixation time (m/s) $p = .000$. According to Mann–Whitney *U*-tests, the AOI referring to the second choice got a higher median score for dwell time recorded as a higher median score of ($Md = 972.91$), the second choice with ($Md = 983.85$) for glance duration, the second choice with ($Md = 988.47$) for diversion duration, the second choice with ($Md = 1042.14$) for glance count, the second choice with ($Md = 1042.14$) for revisits, the second choice with ($Md = 1022.87$) for fixation count and the second choice with ($Md = 972.16$) for fixation time than other AOIs. In other words, the statistically significantly different and higher eye tracking measurements were acquired for the second choice based on the statistical analysis when compared with the eye tracking measurements for the other choices of this question.

5. Discussion, conclusion and implications

The present study was conducted to reveal potential sources of the students' difficulty and misconceptions about geometrical concepts with the help of eye tracking. With this aim,

the students' selection of each inspecting choices and their spending time on selecting particular choices for each question prepared based on the geometrical misconceptions stated in the literature were illustrated by eye tracking and heat maps. The students' selection results based on the choices were explained using descriptive statistics about their misconception. To represent their selection processes in detail and more clearly, eye tracking data were used as suggested in the previous research (Lin & Lin, 2014, 2018; Saß, Schütte, & Lindner, 2017). It was observed that even if the students did not select some choices prepared based on potential sources of misconception, the students might have difficulty and pay attention to these items. To conclude, eye tracking data can provide beneficial and clear information about the students' thinking process of geometrical content, their difficulty and misconceptions. In other words, with the help of eye tracking technology, it becomes possible to illustrate how test takers think about, make selections and progress through the test items by providing information about the process in addition to the test scores. By the eye tracking technology, it became possible to illustrate the cognitive processes of the students (Lin & Lin, 2018) in engaging in answering questions prepared based on geometrical misconceptions. Hence, the potential factors and choices causing students' difficulties could be diagnosed. Eye tracking data are implemented based on the fact that the higher eye tracking scores students acquire and the longer they look at somewhere, the more focused they think about as stated in previous research (e.g. Andra et al., 2015; Hung, 2014; Hyönä, 2010; Rayner, 1998; Kaakinan, Hyönä & Keenan, 2002). Moreover, in this study, it can be stated that the higher eye tracking data score the students get for an inspecting choice for multiple choice and matching geometry problems, the stronger distractor these items become and the more difficulty they have.

When the data collected through think aloud about answering the questions were examined, it was observed that they had difficulty in selecting or eliminating some choices in multiple-choice tests or provided wrong answers. Based on the data referring to the having difficulty in providing (in)accurate answers through think aloud, it could be stated that the students had cognitive conflicts. When the eye tracking data related to the choices for a student were the choices identified based on the criteria that the student had difficulty with while deciding whether they were correct, the data were consistent. In other words, the choices in which the students had difficulty while thinking its accuracy through the think aloud were the choices identified as the parts in which the students had cognitive conflict based on eye tracking data. Therefore, it can be stated that the think aloud process provides information about students' cognitive conflict. The nature of the metacognitive process in the think aloud encourages this statement. The data collected through the think aloud can provide information about the students' cognitive processes including their cognitive conflicts because the think aloud as a kind of metacognitive strategy can provide information about cognitive activities (Berardi-Coletta et al., 1995; Kluwe, 1982; Park, 2004).

As it was observed from the results of the question about the concept of angle and angle measure, approximately half of the students selected the second and fourth choices appropriately. It was observed that the students had difficulty in identifying the angles in equal angle measure through the think aloud. The students had statistically higher eye tracking scores for these items. Most of the students selected these items truly based on the percentage of the students selecting these choices accurately. However, eye tracking data showed that they had difficulty in selecting these items. This conclusion and implication were performed based on the suggestion that higher eye tracking scores such as longer dwell time

or more fixation count can be referred to existence of cognitive processes (Carpenter & Just, 1978; Just & Carpenter, 1976) in the positive relationship with the difficulty of the question or task (Lin & Lin, 2014). Most of the students did not select the fifth choice even if it had been true. Moreover, the AOI representing the question got a higher score from eye tracking measures. It showed that they frequently looked at the question. This can be interpreted that the students had difficulty in determining the angle measure of the angle represented on the question. Also, it can be explained that they paid attention to the question because they could have cognitive conflict in identifying the angle measure and comparing the measures of the angles. Surprisingly, eye tracking data illustrated that the students did not focus and had a cognitive conflict on the fifth choice representing the angle on the question by shortening the half lines (see Figure 1) when the score of this choice was compared with the scores of others. The students could not identify the accurate angles when this angle was represented by a smaller shape and make accurate explanations through the think aloud. It could be stated that the students had difficulty in identifying the angles in equal angle measures. The difficulty of the students having misconceptions and difficulty in angle and angle measure are in line with the findings of the previous research (Devichi & Munier, 2013; French, 2004). In this respect, it can be concluded that the angle and angle measure concept should be taught by using different representations of similar angles and in different orientations. Moreover, the smaller representations of the angles in equal angle measure in addition to their bigger ones should be provided to the students.

The findings of the question about the concept of the triangle represented that the students could easily identify the triangles illustrated by typical examples in different orientations. However, even if it was observed that most of the students could select the third choice truly (see Figure 2), think aloud and eye tracking data showed that they focused and had a cognitive conflict on this choice. In this choice, the figure looks like a triangle, but it is composed of four line segments. The reason for this situation can be stated in a way that the use of typical examples can keep the students from paying attention to the critical attributes of the triangle as it has been explained in the previous research (Tsamir, Tirosh & Levenson, 2008; Tsamir et al., 2014). In line with this view, it can be stated that the teachers should insist on critical attributes and use typical and non-typical examples together while teaching triangles. Moreover, the students should be provided an examination of these attributes on these examples by criticizing.

The findings of the question about the altitude of triangles showed that the students could recognize the altitude of acute triangles and right triangles when they were represented by typical examples as it was done on the second and fourth choices. Most of the students could identify the wrong and accurate representations of the altitudes of obtuse-angled triangles appropriately. Through the think aloud, they could not make explanations easily and immediately. Conversely, even if it was observed that most of the students could make the selections truly, eye tracking data showed that they focused and had a cognitive conflict on the first and third choices representing obtuse angle triangles (see Figure 3). The reason for this situation can be stated in a way that the use of typical examples can keep the students from paying attention to the formation of altitudes of obtuse triangles and the students could not have sufficient understanding to obtuse-angled triangles' altitudes as it has been explained in the previous studies (Alatorre & Saiz, 2009; Hershkowitz, 1990; Tsamir, Tirosh & Levenson, 2008; Tsamir et al., 2014; Uygun & Akyuz, 2019). Surprisingly, it was observed that most of the students identified the sixth choice as true inaccurately

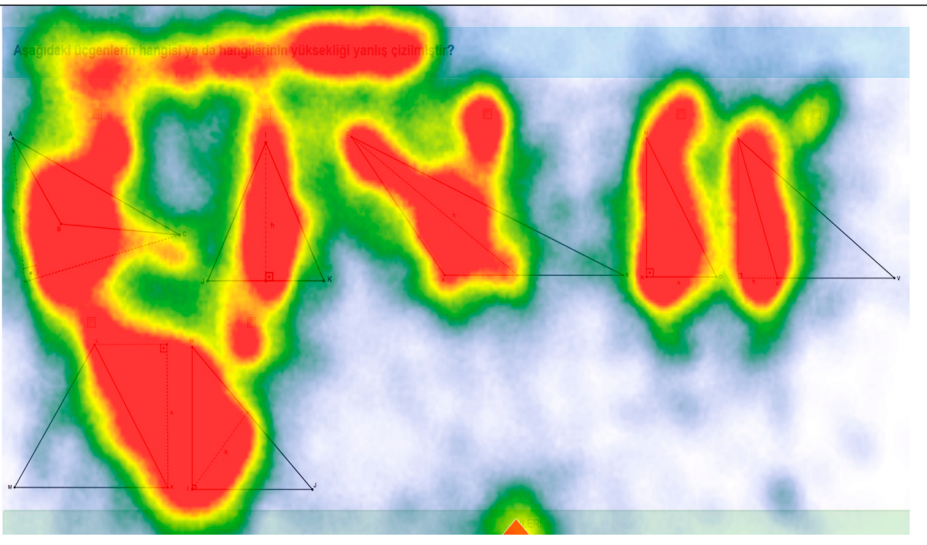
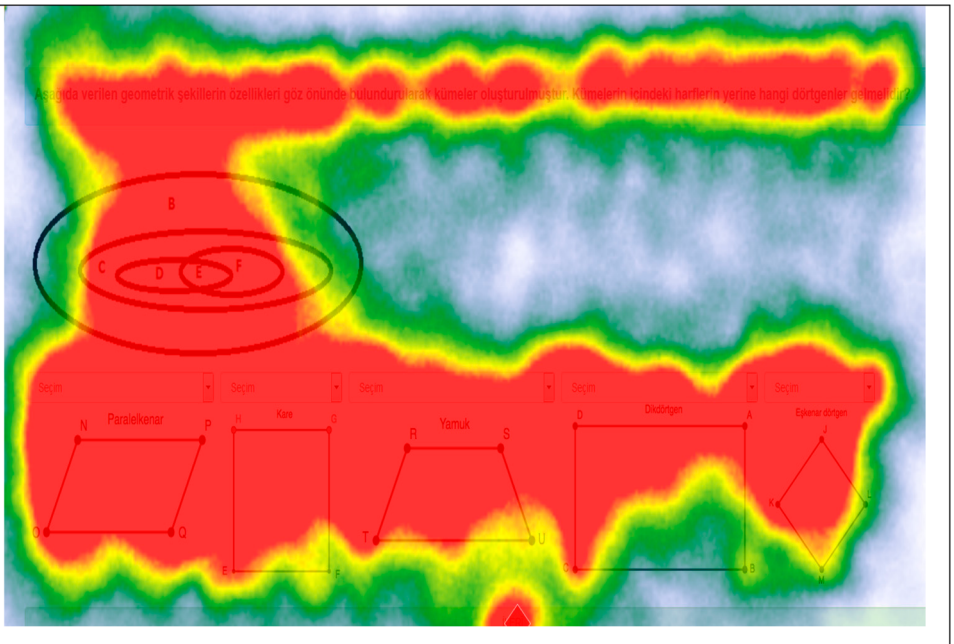


Figure 3. Heat map for the question about the altitude of triangles.

(see Figure 3). In this choice, it is an acute-angled triangle, and there is altitude formed by moving the accurate altitude outside the triangle. Eye tracking data also showed that they paid attention to this item less than other choices. In other words, most of the students accepted this choice as true without insisting on it. Based on this result, it can be stated that the students can over generalize the knowledge about the formation of altitudes outside of the obtuse-angled and right triangles. Hence, it is important for them to learn the formation of triangles' altitudes by examining different types of triangles. Furthermore, the altitudes that belonged to all sides of a triangle should be examined in various triangles in the lessons (Figures 4–6).

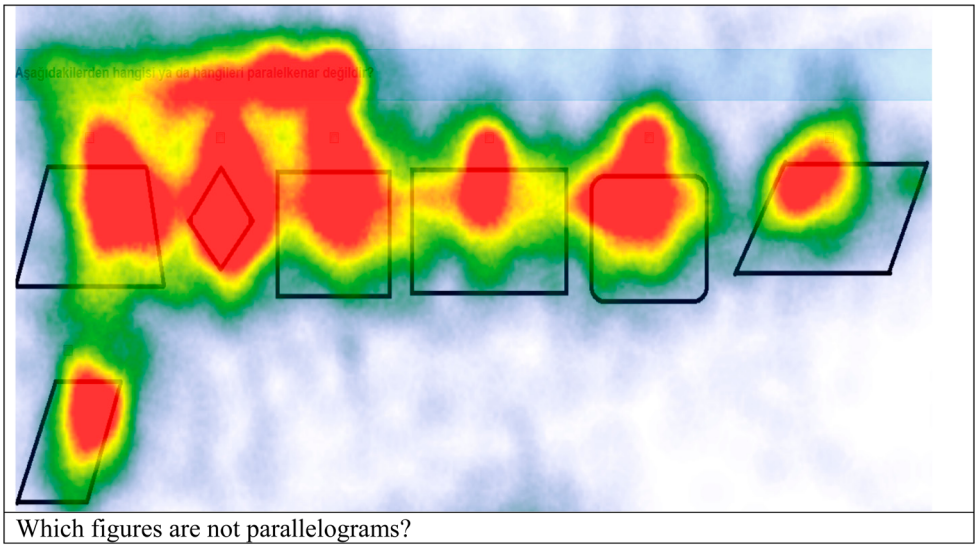
The findings of the question about the hierarchical relationship among quadrilaterals represented that the students could not place them on a Venn diagram based on their hierarchical connection. Through the think aloud, they had difficulty in making explanations and they could not tell their thoughts about the placement of quadrilaterals based on their hierarchical relationship immediately. Their matching the letters with the quadrilaterals is in line with the eye tracking data of the process of answering this question. In other words, approximately 20% of the students, only a few students could accurately identify these relations among quadrilaterals. Moreover, eye tracking data showed that they paid attention and had a cognitive conflict on the Venn diagram (Figure 4). The heat map also represented this finding clearly. In this respect, it can be stated that the students had difficulty in identifying these connections, and most of them could not answer the question accurately. Moreover, this finding can be confirmed by previous research studies (Jones, 2001; Monaghan, 2000; Pickreign, 2007; Erez & Yerushalmy, 2006).

To specialize the connection among quadrilaterals, a question focusing on parallelograms as a type of quadrilateral was asked to the students. It was observed that the students could easily identify the typical examples as in the sixth and seventh choices (see Figure 5)



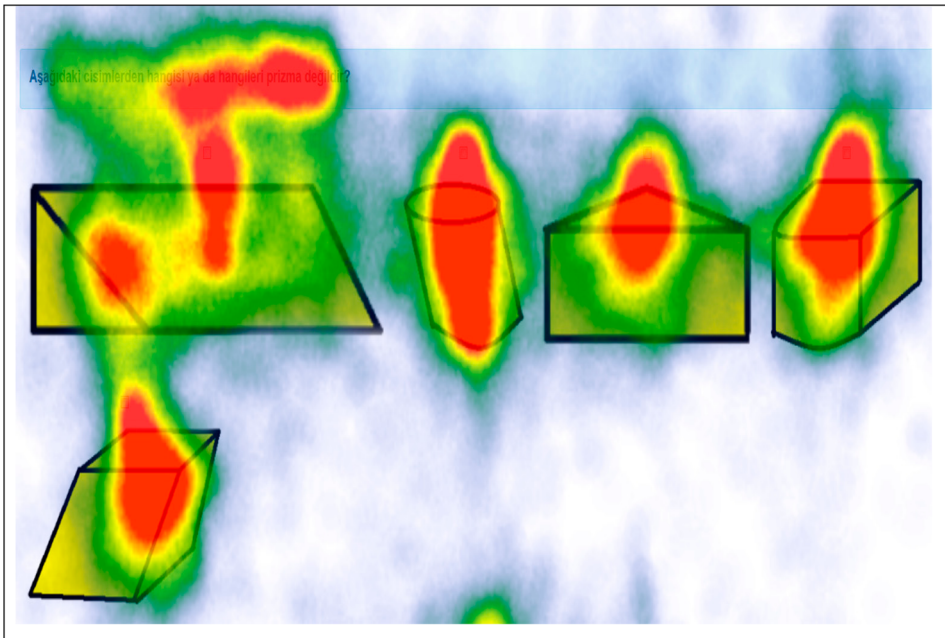
The following Venn diagram is formed based on the properties of types of quadrilaterals. Each letter representes a type of quadrilateral. Please, match the letters with the quadrilaterals.

Figure 4. Heat map for the question about the quadrilaterals.



Which figures are not parallelograms?

Figure 5. Heat map for the question about parallelograms.

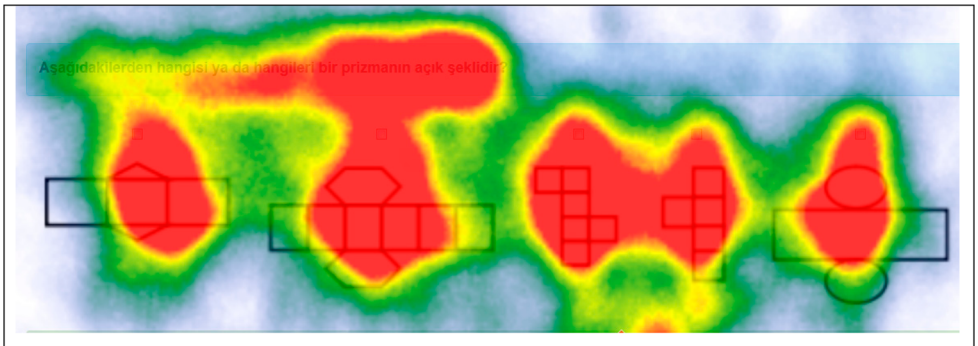


Which figures do not represent prisms?

Figure 6. Heat map for the question about the prisms.

and make explanations immediately through think aloud. Furthermore, eye tracking data showed that they did not have a cognitive conflict in determining these typical examples. Also, they could identify the critical attribute of connection of four line segments forming a closed shape easily and did not have cognitive conflict as in the fifth choice as a closed curve. Data showed that most of the students could determine that 'trapezoid is not parallelogram' and 'square and rhombus are parallelograms', and eye tracking data showed that the students had a cognitive conflict and paid attention to rhombus and squares in different orientations. Moreover, think aloud data supported this finding since some of them could not produce explanations about rhombus and squares in different orientations immediately. They explained by thinking about them longer than other choices, and some of the students changed their minds more than twice while deciding the accuracy about these choices. The students' difficulty about the identification of the relationship among parallelograms, rhombus and square can be encouraged by the studies by Okazaki and Fujita (2007) and Ay and Basbay (2017). In this respect, it can be stated that rhombus and square figures are strong distractors for this question. Hence, the teachers should insist on the connection between these quadrilaterals and parallelograms in their lessons.

The findings of the question about the prisms represented that most of the students could easily identify the prisms and not prisms in the third (triangular prism), fourth (cylinder) and fifth (oblique prism) choices accurately (see Figure 6). Through the think aloud, they could produce explanations about these choices immediately and easily. Also, the data showed that most of the students determined the cylinder as prisms. Eye tracking data also showed that the students had difficulty in determining whether the cylinder was



Which figures represent opening forms of prisms?

Figure 7. Heat map for the question about the opening forms of prisms.

a prism. Moreover, most of the students identified the fourth choice as not a prism accurately. However, eye tracking data showed that they paid attention and cognitive conflict on thinking about this choice. Also, the students could not explain the hierarchical relationship between prism and cylinder easily and immediately through the think aloud. This might result in that the students had inadequate knowledge about the hierarchical relationship between cylinders and prisms and 'prisms are always cylinder but all cylinders are not prisms'. The students' difficulty about the identification of prisms and cylinders is consistent with the findings of the previous research (Accascina & Rogora, 2006; Koçak, Gökkurt & Soylu, 2014; Linchevski, Vinner & Karsenty, 1992). Moreover, based on the findings of the question about the opening form of the prisms, nearly half of the students could accurately identify these forms accurately (Figure 7). Eye tracking data showed that they had more difficulty and cognitive conflict on the second choice. This case can be explained in a way that the increase in the number of the sides of the prisms might make the students to have difficulty and cognitive conflict. Hence, it can be stated that the prisms having more sides used in typical examples are strong distractors. Furthermore, most of the students selected the opening form of the cylinder as the opening form of the prism inaccurately. Eye tracking data showed that the students made this mistake without paying more attention to and having cognitive conflict. In this respect, it can be stated that they were not aware of the hierarchical relationship between cylinders and prisms. Similarly, in the previous question, this might result from the students' inadequate knowledge about the content. In this respect, it is important to insist on the critical attributes of prisms and the hierarchical relationship between cylinders and prisms while teaching prisms to the students. Also, it is important to have students engage in various examples of prisms in the lessons.

The findings of the current study represented the cognitive conflicts that the students had in thinking about the geometrical concepts. Hence, it was observed that the eye tracking and the think aloud data could provide information about the students' cognitive conflicts related to their geometrical misconceptions. Hence, the present study can make contribution to educators and teachers by emphasizing and showing the importance of the relationship between visual attention and cognitive conflicts and by providing detailed information about students' geometrical misconceptions. On the basis of the findings of the

current study, teachers can be advised that while identifying students' geometric misconceptions, the answers given by the students to the questions are not sufficient. The students' visual attention and the think aloud process should be considered to acquire detailed, correct and clear information about their geometric misconceptions. In identifying students' geometrical misconceptions, the teachers should pay attention to how their students analyse the questions, how long they think about them, which parts they focus on and how quickly and easily they could produce explanations about their answers.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendix

(1) Geometry test
http://kisaca.site/fKJjW
(2) A video recording example of answering geometry test made by eye tracking tool
http://kisaca.site/L0lW3.