Analysis of Structures with Unilateral Boundary Conditions

Gebrail Bekdaş*, Rasim Temür* and Y. Cengiz Toklu[1]

Abstract
Unilateral boundary conditions are imposed on structures for example when the supports are one sided like tensionless foundations. These kinds of supports may be generalized by considering unsymmetrical supports where the supports behaviour depends on the sign of the support reaction. Such behaviour makes the structure nonlinear and increases the difficulties in their analysis. Current methods of structural analysis can be applied to these kinds of problems with important modifications involving predictions and corrections without a general solution. A general way of solving these problems can be obtained through the application of the method called Total Potential Optimization using Metaheuristic Algorithms (TPO/MA) which is launched very recently. In this study examples are given for this kind of solutions.

Keywords: Unilateral Boundary Conditions, truss structures, energy methods, harmony search, TPO/MA.

1 Introduction

Mathematical models of the traditional analyses methods, boundary condition of the nodes and supports is described linearly and angularly. Generally, these conditions are defined along the direction, i.e. x-x, y-y, and the condition changing according to direction are not taken into consideration.

In Fig. 1, a column footing under the axial force N and flexural moment M is shown. The mathematical model of the footing is also given in Fig. 1. In this model, soil is defined by the help of springs. Suppose that, ground reaction forces are formed as compressive forces between A-B, and as tensile forces between B-C.

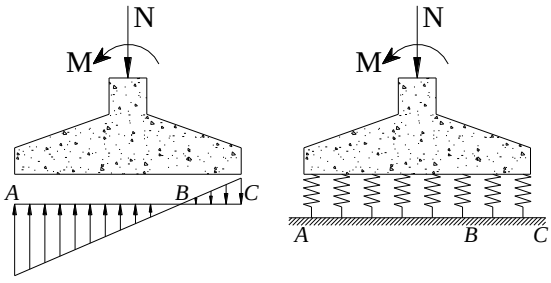

Fig. 1. Column footing and its mathematical model

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As it is known, the ground reactions can be considered to consist only of compressive forces as shown in Fig. 2. In that situation, mathematical model must be rearranged according to reactions coming from springs (Fig. 2). This soil-structure interaction behaviour can be given as an example for unilateral boundary condition.

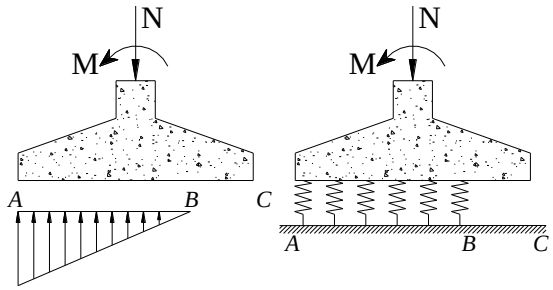

Fig. 2. Rearranged forces and model

In the classical approaches, unilateral boundary condition behaviour is taken into consideration by performing iterative analyses. In these analyses, systems is analysed firstly. Then, results are evaluated. If there is a contradiction, such as tensile forces on soil, the boundary condition of the nodes is rearranged. After that the analyses are repeated. This iterative approach continues, until all boundary condition is satisfied.

In this study, the analyses of truss structures which contain unilateral supports are investigated. The analyses are performed based on TPO/MA [1, 2, 3] technique. In the present application of TPO/MA, the Harmony Search (HS) method [4] is used as metaheuristic algorithm. The main advantage of this technique is to do analyses of systems without any additional process as it must be done traditional ones. The technique is performed on two examples and results are compared with finite element method (FEM). According to analyses result, the proposed technique is effective on solving truss structures by considering unilateral boundary conditions.

2 Methodology

A music inspired metaheuristic algorithm Harmony search algorithm developed by Geem *et al.* [4] is employed for TPO/MA optimization. According to HS algorithm rules the optimization process can be summarised in five steps.

Step 1: In the stage the design constrains including material properties and cross-sectional dimensions of truss member, coordinates (unloaded) and boundary condition of joints, start and end joints for each truss and loading condition of the system are defined. Also, a solution range (r) of joints and parameters of HS algorithm, harmony memory size (HMS), harmony memory considering rate (HMCR) and pitch adjacent rate (PAR) is entered.

Step 2: The initial harmony memory (HM) matrix is generated. This matrix is constructed by using harmony vectors (HVs). The number of these vectors is equal to HMS. Harmony vectors contain randomly generated coordinates of the joints with in the solution range. In Fig. 3, an example for solution range of a 3- joint truss is shown. The figure shows the possible coordinates of the joints 1, 2 and 3 as the shaded areas A_1, A_2 and A_3, respectively. This area is defined by entering radius (r) of the circle.

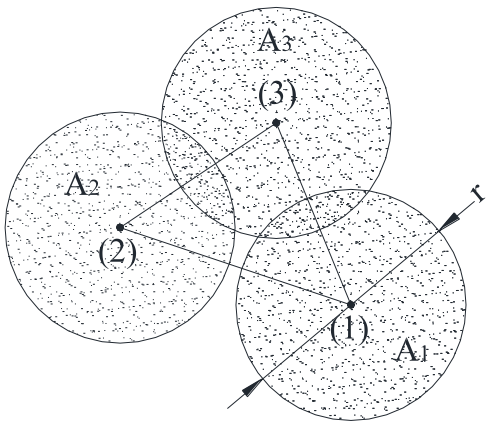

Fig. 3. Solution range

Step 3: For each HV the strain energy (SE), external forces energy (EFE) and total potential energy (TP) of the system are calculated by using the coordinates generated in Step 2. The details of the this procedure is given elsewhere [1, 2, 3]. The main objective of the optimization is to find coordinates of each joints (deformed shape of the system) where TP value of the system assumes a minimum. Thus, the TP values of the HVs is stored in HM for future comparisons.

Step 4: In this step, a new harmony vector is generated according the special rules of the HS algorithm. This vector can be generated in two ways. In the first way, this vector can be generated from whole solution range as it previous ones. In the second way, this vector is generated by using one of HV stored in HM. In that situation the coordinates of vector is generated from smaller range around the existing vector coordinates. This smaller range is defined by multiplication of the whole solution range dimension (r) and PAR value [2,4]. The related decision is taken by looking at HMCR value. HMCR is described as the possibility of generation of the vector existing vector. After the generation of new vector, TP value of the vector is calculated and it is compared with existing vectors energies. If the new vector energy is less than the worst existing vector (the vector has maximum TP), the worst vector is replaced with new one.

Step 5: The stopping criteria is checked. This criteria is defined the number of the iteration. If the stopping criteria is satisfied the optimization process is ended and if not the process is continue from the Step 4.

3 Numerical Example

The proposed analysis by TPO/MA method is applied on the 21-bar truss structure shown in Fig. 4. The cross-sectional areas and elasticity modulus of all members are 100 mm^2 and 200000 N/mm^2, respectively. The width of each span is 3.0 m and the height of the structure is 4.0 m. A, B and C supports have unilateral conditions which are free in the upper direction (+y direction) and fixed in the down direction (-y direction). Also, these supports are free in x-x direction and the support D is hinged (see Fig. 4).

The system is analysed under 10 cases with different loading condition (see Table 1) in order to investigate unilateral support condition behaviour step by step. The intensity of P and R loads are 10000 kN.

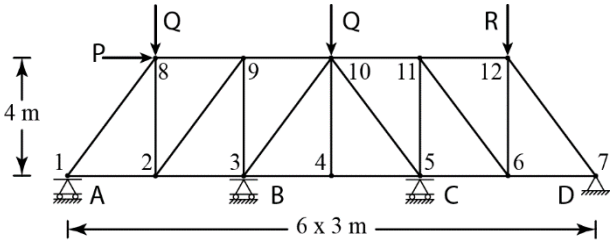

Fig. 4. 21-bar truss structure

Table 1. Load cases

Case	Q [N]	Case	Q [N]
1	-5000	6	-2500
2	-4500	7	-2000
3	-4000	8	-1500
4	-3500	9	-1000
5	-3000	10	-500

The deformed shape of the structure for the cases can be seen in the Fig. 5. As seen in the figure, by decreasing load intensity of Q, first in the support A (for the Cases 5 to 10) then in the support B (for the Cases 9 and 10) displacements (+v) in upper direction is observed.

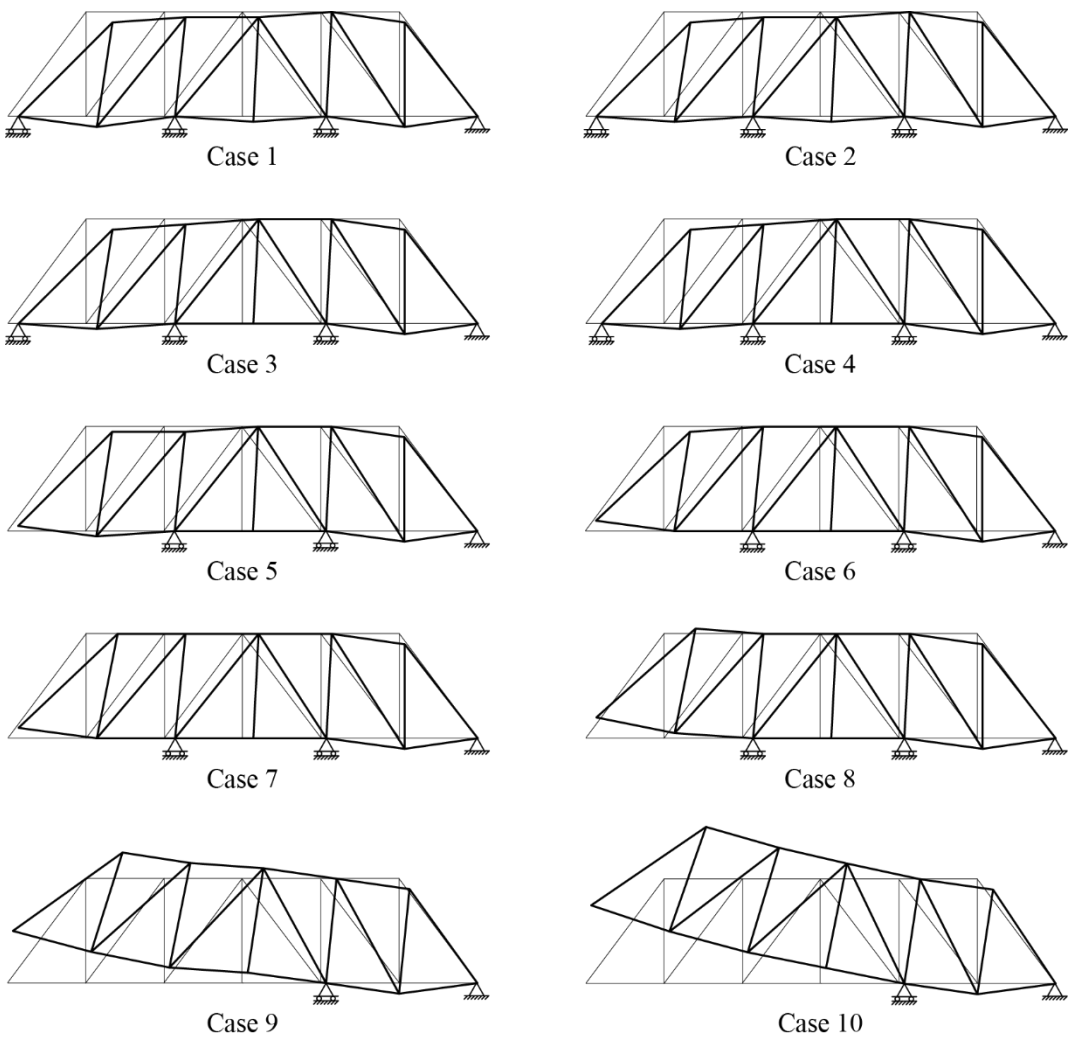

Fig. 5. Deformation of the example truss system (Scale factor for displacements=200)

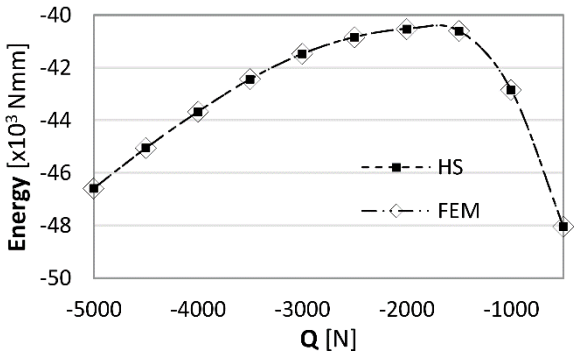

Fig. 6. Total potential energy of the system for different loads.

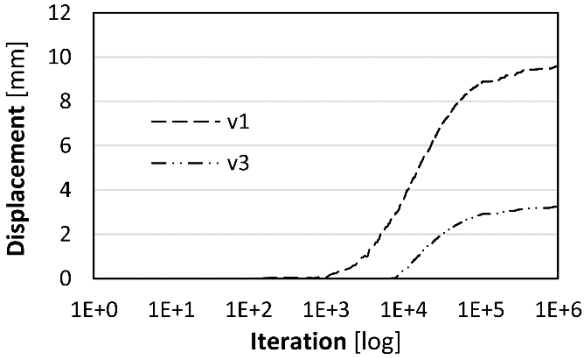

Fig. 7. The y direction displacement vs iteration number of joints 1 (A support) and 3 (B support) (Case 10)

In Fig. 6, total potential energy of the system obtained from TPO/MA and FEM are plotted. It can be seen from the graph that in all load cases the energy values of the methods gives similar results. The y direction displacements vs. iteration number for joints 1 (unilateral support A displacement, v_1) and 3 (unilateral support B displacement, v_3) can be seen in Fig. 7, corresponding to loading case 10. The figure indicates that at the beginning of the iterations both displacements are zero. As the iterations continue, first at the support A and then at support B displacements take place in +y direction indicating that these supports are in fact inactive under this case which corresponds to loading 10. The changing of the energy values stored in harmony matrix during the optimization is given in Fig. 8. At the figure, best and worst solutions are

defined according to objective of the optimization. The figure indicates clearly the diminishing of the gap between the best and the worst values, and also the approach to a state where the total potential assumes a minimum.

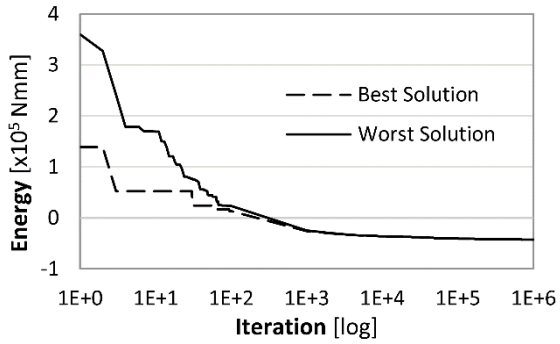

Fig. 8. Energy vs iteration number for the best and worst solutions for loading case 10

4 Conclusion

In the study, the unilateral support condition is investigated by using TPO/MA technique. The main advantage of the proposed method, the support conditions structural system can be easily defined. Thus, the analyses are performed automatically without user intervention. This is not the case for FEM solutions where it is required from the user the determination of the supports with zero reaction. It is obvious that with the increase of the number of one-sided supports and with more complicated loads applications with FEM will be beyond the reach of a user.

The proposed method is performed a 21 bar truss system under different load cases. Same analyses are also done with finite element method. Analyses results show that, the energy values of proposed method (TPO/MA) give compatible results with finite element method.

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