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FUZZY CONJOINT MODEL FOR PRESERVICE MATHEMATICS TEACHERS' BELIEFS

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Abstract

The purpose of this paper is to measure the beliefs of preservice mathematics teachers' about mathematics using a fuzzy set. In this study, the beliefs of preservice mathematics teachers related to the nature of mathematics and learning mathematics were questioned. The seven attributes have been used to measure preservice mathematics teachers' beliefs about mathematics. The data were analyzed by fuzzy conjoint analysis. Preservice mathematics teachers' beliefs have been recorded in form of degrees of similarity and its level of agreement. The results showed that preservice mathematics teachers' beliefs varied. This diversity is indicated by the agreement of the different levels and the similarity ratings of the measured features.

1. Introduction

1.1. Beliefs and mathematics education

A careful reading of the early psychological literature reveals that beliefs

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and belief systems began to be explored in the beginning of 20th century, particularly by social psychologists. However, as the more behavioral perspectives of learning attracted increased attention, research endeavors turned to the more readily observed parts of human behavior, with a consequent loss of interest in beliefs. With new developments in cognitive science in the 1970s, attention to beliefs and belief systems re-emerged. Work on beliefs can be found in areas as diverse as political science, history, psychology, sociology, anthropology, and education (Leder et al. [15]).

Leder and Forgasz define belief as: In everyday language, the term "belief" is often used loosely and synonymously with terms such as attitude, disposition, opinion, perception, philosophy, and value. Because these various concepts are not directly observable and have to be inferred, and because of their overlapping nature, it is not easy to produce a precise definition of beliefs (Leder and Forgasz [14]).

Today, employees with problem-solving and analytical thinking skills are needed.

This need is also clearly seen in the field of education. In mathematics education, it is not sufficient for the students to learn only operational mathematics knowledge. It is also foreseen that students should have skills in mathematical thinking and problem solving methods. Teachers' beliefs are very important in the development of mathematical thinking skills of students. Mathematics teachers' beliefs about the nature of mathematics and learning mathematics can change the negative opinions of students against mathematics (Smith [20]). Therefore, it is necessary to understand the beliefs of preservice mathematics teachers' (PMT).

Beliefs play great role in mathematics learning and teaching. The learning outcomes of students are strongly related to their beliefs and attitudes about mathematics (Furinghetti and Pehkonen [9]). Thus assessing or evaluating of students mathematical knowledge must be made in awareness of their beliefs. A very important part of the beliefs about mathematics occurs during childhood and school life (Frank [8]).

Success in solving mathematical problems is based not only on having knowledge about mathematical content but also on other factors such as decisions, strategies, emotions and beliefs used. These concepts are not considered much in mathematics education. But these concepts affect how people think, how they approach the problems and how they act. At the same time, these concepts are also important in terms of how people work on mathematics and when they focus (Garofalo [10]).

It is necessary to understand the beliefs of mathematics teacher candidates and mathematics teachers about what mathematics are, how to know mathematics, how to do it, and what the teacher means. Researchers working on it indicate that these beliefs may be factors in maintaining the teaching of mathematical ideas (Smith [20]). In addition, various researches have shown that the beliefs about the nature and learning of mathematics affect the mathematics education of the students (Thompson [25], Carter and Norwood [4], Steele and Widman [23], Baydar and Bulut [2]). It is very important to be able to assess teachers' beliefs for improve mathematics teaching. In recent years, many articles have been published about the importance of teacher belief in mathematics.

Thompson (Thompson [25]) has shown that teachers' beliefs, perspectives, and preferences, both consciously and unconsciously about the nature and teaching of mathematics, play an important role in shaping attitudes and behaviors at the teaching stage. In addition, teachers' attitudes and behaviors at the teaching stage are a reflection of their beliefs. According to this, every effort to improve the quality of mathematics education must be understood that teachers' beliefs and these beliefs are influential on teachers' movements. If the important role of teachers' beliefs about the nature and teaching of mathematics is ignored, then, the work to be done in improving the quality of mathematics education will be misguided and incomplete.

It is possible to give examples of how teacher behaviors are influenced by teacher beliefs. For some mathematics teachers, mathematics is an art such as poetry, music, painting. For another group, mathematics is a well-defined set of information. For some, mathematics is a way to explain the

world. These different thoughts influence decisions about what to teach. It also influences how subjects should be taught.

1.2. Fuzzy analysis

Since 1965, the fuzzy theory as a new theory introduced by Zadeh in his celebrated paper (Zadeh [28]), has generated a growing interest both in the theoretical mathematical framework as well as in its practical applications. A fuzzy set approach has been developed to solve problems in which the descriptions of activities are imprecise, vague and uncertainty. The applications of fuzziness are uncountable and varied. The fuzzy literature is vast and huge and goes to all possible directions. Fuzzy sets were taken up with enthusiasm by engineers, computer scientists and operations researchers. A notable reason is the relationship that fuzzy sets have with multivalued logic, offering decision possibilities such as ”may be true” and ”may be false”, suitably quantified, in addition to the traditional dichotomy of ”true” and ”false”. Vast, interconnected arrays of such decisions can be processed rapidly by computers.

Fuzzy sets have a great progress in every scientific research area. It found many application areas in both theoretical and practical studies from engineering area to arts and humanities, from computer science to health sciences, and from life sciences to physical sciences. In the recent years, ordinary fuzzy sets have been extended to new types and these extensions have been used in many areas such as energy, medicine, material, economics and pharmacology sciences.

According to Alsina and Trillas [1], the scope of mathematics education must offer an attractive panorama of all procedures, concepts, descriptions, and representations which may be useful for future generations. Exact reasoning and approximate reasoning are both useful if they follow some rules of correctness. Thus, fuzzy logic are considered as powerful tools for modelling, mathematical, many interesting facts integrating aspects of human knowledge.

In classical, or crisp, sets the transition for an element in the universe between membership and non-membership in a given set is abrupt and well defined. For an element in a universe that contains fuzzy sets, this transition can be gradual. This transition among various degrees of membership can be thought of as conforming to the fact that the boundaries of the fuzzy sets are vague and ambiguous. Hence, membership of an element from the universe in this set is measured by a function that attempts to describe vagueness and ambiguity.

Let X be the universe of discourse and its elements be denoted as x. In classical set theory, crisp set A in the universal set X is a collection of well-defined objects and is characterized by the function $f_A(x) : X \rightarrow \{0, 1\}$ such that $f_A(x) = 1$ if $x \in A$ and $f_A(x) = 0$ if $x \notin A$. This set maps universe X to a set of two elements. For any element x of universe X, characteristic function $f_A(x)$ is equal to 1 if x is an element of set A, and is equal to 0 if x is not an element of set A. Zadeh proposed the idea of a fuzzy set as expansion of the classical set theory to deal with uncertainties (Zadeh [28]). In the fuzzy theory, fuzzy set A of universe X is defined by function $\mu_A(x)$ called the membership function of set A, $\mu_A(x) : X \rightarrow [0, 1]$, where $\mu_A(x) = 1$ if x is totally in A; $\mu_A(x) = 0$ if x is not in A; $0 < \mu_A(x) < 1$ if x is partly in A.

Conjoint analysis also called "trade-off analysis" helps to fathom how people make complex decisions. The basic assumption underlying the technique is that complex decisions such as purchase decisions are based not on a single factor or criterion, but on several factors known as considered jointly (Bhattacharyya [3]). Smith [21] has defined conjoint analysis as a methodology for the measurement of psychological judgments, such as consumer preferences. Stimuli in the form of product configurations, advertisements, etc. are presented to the respondent for evaluation. Conjoint analysis originated out of the mathematical psychology research of conjoint measurement. Green and Wind [11] has stated that conjoint measurement is concerned with measuring the joint effect of two or more independent

variables on the ordering of a dependent variable. Conjoint analysis attempts to jointly identify the composition model for decision choices and at the same time estimate the utility value of the attributes that are important in the choice decision.

The fuzzy set conjoint model is purely based on the fuzzy sets preference model. The measurement of each attribute was presented in the form of degree of membership which is believed to be able to reflect the degree of perceptions. The fuzzy set conjoint model can be adapted into the metrics to ease the users ability to evaluate the usability by using natural language, which is full of vague and subjective expressions.

Fuzzy conjoint analysis has been employs in many studies which involving the use of Likert-scale to represent linguistic terms. It is believed that the fuzzification of this Likert-scale is more representing the vagueness content on human subjectivity in giving judgment. Rasmani and Shahari [18] applied the fuzzy analysis for evaluating job satisfaction amongst academic staff in a higher education institution. Their study has shown that the analysis using the fuzzy approach gave very good results and provides more information than the statistical analysis based on percentage. Fuzzy conjoint analysis has also been applied in research in education sector. Study done by Lazim and Osman [13] is one of the studies in the area. Their study was focused on the application of a fuzzy set in measuring teacher's beliefs about mathematics, and it has showed that using fuzzy set for the research has lead them to a useful conclusion concerning the strength of teacher's beliefs about mathematics.

The precision of formal languages contrasts rather sharply with the imprecision of natural languages. To reduce the gap between them, it is natural to introduce randomness into the structure of formal languages, thus leading to the concept of stochastic languages (Lee and Zadeh [16]). Another possibility lies in the introduction of fuzziness. This leads to what might be called fuzzy languages. It appears that much of the existing theory of formal languages can be extended quite readily to fuzzy languages. The fuzzy linguistic variable is a variable that reflects the different levels of human language with a range from natural to artificial languages (Zadeh [30-31]).

The linguistic variables related to a human word or statement can be categorized into numerous linguistic criteria such as equally important, moderately important, strongly important, very strongly important, and extremely important.

Similarity means that the notion is defined a generalization of the concepts of equivalence (Zadeh [29]). Although the concept of similarity is a basic concept in human cognition, there does not exist a valid, general-purpose definition of similarity. There do exist many special-purpose definitions which have been employed with success in cluster analysis, search, classification, recognition and diagnostics. There are several similarity measures that are proposed and used for varied purposes. Similarity is a core element in achieving an understanding of variables that motivate behavior and mediate affect. Thus, the concept of similarity is of fundamental importance in applied areas. Since the concept of similarity has a wide range of applications, there are different approaches present in literature as axioms for degree or measure of similarity. These axioms have differences and similarities depending upon the contexts in which they are constructed. Therefore, the concept of similarity measure is used in a variety of scientific fields such as decision making, pattern recognition, machine learning and market prediction. Fuzzy set theory has developed its own measures of similarity, which applications in various scientific areas.

This paper describes the evaluation of (PMT) beliefs based on the application of fuzzy set theory. Fuzzy set theory provides a mathematical model in which vague and subjective expressions can be precisely quantified. Then, in this paper, the integration of fuzzy logic and conjoint analysis is used for evaluation about mathematics nature and learning of mathematics beliefs of PMT.

2. Methods

In this study which is a descriptive analysis, a survey model was used. With the help of the Likert type surveys prepared for this purpose, the opinions of the participants were determined.

In education and social sciences researches, various measurements are conducted. Psychometric instruments are used in these researches. The most fundamental and frequently use of psychometric tools is the Likert scale. Likert scale was developed by Dr. Rensis Likert in 1932. Rensis Likert intended to develop a tool to measure “attitude” scientifically.

The original Likert scale is a set of statements (items) offered for a real or hypothetical situation under study. In this scale, participants are asked to show their level of agreement (from strongly disagree to strongly agree) with the given statement (items) on a metric scale.

Likert-type questions include a statement of attitude or opinion on the topic being searched, and options indicating the level of participation in that statement. In the case of Likert-type questions, multiple options are provided between two extremes to determine the level of participation. These options are ranked from ”highest to lowest” or ”best to worst”. A Likert scale which has an odd number includes a midpoint. Generally, Likert scale consists of 5-7 items. 7 items are better than 5 and 9 items to achieve measurement of a continuous variables. This is because the sensitivity of the measurement will be increased and be more confident of having a continuous rather than a categorical variable.

In this research, the Likert scale is used for all preference ratings, providing subjects with a balanced selection of 7 linguistic terms. This scale has a central neutral evaluation with three positive and three negative evaluations of increasing severity, and is used only as an ordinal scale. The linguistic terms and their numeric labels are: very strongly agree (1), strongly agree (2), agree (3), neutral (4), disagree (5), strongly disagree (6) and very strongly disagree (7).

The following questions were prepared. The research questions were asked to the preservice mathematics teachers:

(1) Mathematics is a discipline that includes abstract concepts.

(2) Mathematics is accepted as a difficult and frightening knowledge domain.

(3) For real life, mathematics is very important.

(4) Reasoning, hypothesizing and logical deduction are the products of the human intelligence and these point out the basic principle of mathematics.

(5) Mathematics is the center of physical world or truths of the universe.

(6) It is the only way to learn mathematics to understand its conceptual meaning and to perform an application.

(7) Mathematics is the biggest shared value of human brain. So, universality is the power of mathematics.

In this study, PMTs who have just completed the last grade of the Department of Mathematical Education of Istanbul University have been selected. Because PMTs completed mathematics courses and pedagogical formation program successfully for 4 years. Thus, the participants have knowledge about the nature of mathematics and learning mathematics.

Smithson [22] published the first comprehensive introduction of fuzzy set analysis for behavioral and social sciences. Following the Smithson [22], some researchers have tried to extend the application of fuzzy sets to various fields in social sciences. The fuzzy rating scales have been used to measure educational achievement by Fourali [7].

By using the model of Turksen and Willson [26], this work focuses on an evaluation of the PMT beliefs about mathematics. This model has been considered in many studies which involving the use of Likert scale to represent linguistic terms (Lazim and Abu Osman [13], Rasmani and Shahari [18], Mohd-Tawil et al. [24]). Turksen and Willson [26] applied this model in investigate customers preferences and satisfaction on products and services. Their fuzzy set conjoint model has also been applied to education. Lazim and Abu Osman [13] had studied on the application of fuzzy set in measuring teachers' belief about mathematics. They showed that use of fuzzy set for the research led them to see useful conclusion concerning the strength of teachers belief about mathematics.

Furthermore, Rasmani and Shahari [18] applied this model for evaluating job satisfaction amongst academic staff in a higher education institution. Their study showed that the analysis using the fuzzy approach gave good results and provided more information than the classical statistical analysis based.

In this work, for each attribute M, the calculation of degree similarity between the fuzzy set representing the whole respondents with every fuzzy set that represented by seven linguistic values $(Q_n,\ n = 1, 2, 3, 4, 5, 6, 7)$ was executed using Euclidean distance formula.

2.1. Method and algorithm

The set Q_n has the linguistic variables such as very strongly agree, strongly agree, agree, undecided, disagree, strongly disagree, very strongly disagree where $n = 1, 2, 3, 4, 5, 6, 7$. The fuzzy sets represented for each linguistic value Q_n were defined as is the following table (Lazim and Abu Osman [13]):

Table 1. Linguistic value Q_n for $n = 1, 2, 3, 4, 5, 6, 7$

Q_n / n	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Q_1	0.8	0.5	0.2	0	0	0	0
Q_2	0.7	1	0.6	0.4	0	0	0
Q_3	0.4	0.6	1	0.6	0.4	0	0
Q_4	0	0.3	0.7	1	0.7	0.3	0
Q_5	0	0.2	0.4	0.6	1	0.6	0.4
Q_6	0	0	0	0.4	0.6	1	0.7
Q_7	0	0	0	0.2	0.5	0.8	1

Let R be a fuzzy set, M be a factor attributes. For ith respondent $(i = 1, 2, 3, ..., 60)$ against attribute M,

$$R_i \in \{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7\}.$$

Denote $\mu_{R_i}(x_j, M)$ degree of membership of domain element x_j in the subject's linguistic rating *R* of the *i*th attribute of *M* for each element in fuzzy set R_i, where $x_j = 1, 2, 3, ..., l$. l is a number of linguistic values used and in this study $l = 7$.

The approximate degree of membership for each element, y_j, $(j = 1, 2, 3, ..., 60)$ in fuzzy set *R* is defined as

$$\mu_R(y_j, M) = \sum_{i=1}^{n} W_{i_R_i(x_j; M)}, \tag{1}$$

where *n* is a number of respondents, $W_i = \dfrac{w_i}{\sum_{k=1}^{n} w_k}$ and w_i is a score of linguistic values given by *i*th respondent.

The formula for the similarity of two sets is

$$(Q_n) = \frac{1}{1 + \sqrt{\sum_{j=1}^{7} (\mu_R(j, M) - \mu_{Q_n}(j))^2}}. \tag{2}$$

This formula was developed by Turksen and Willson [26] using Euclidean distance between fuzzy sets. In equation (2), $\mu_{Q_n(j)}$ is the fuzzy set defined for linguistic value and $\mu_R(j, M)$ is the calculated set for attribute *M* from equation (1).

The algorithm for measuring similarity is as follows:

Algorithm:

Step 1. Take answers of PMT.

Step 2. Calculate the score of linguistic values for each respondent.

Step 3. Calculate weights for each answer with W_i.

Step 4. In equation (1), find the membership of each element in fuzzy set *R*.

Step 5. In equation (2), compute the similarity of R and Q.

Step 6. Select the linguistic value of Q_n, $(n = 1, 2, ..., 7)$ which founded the highest degree of similarity.

3. Results and Discussion

In this section, each question asked to PMT has been evaluated, presented and interpreted separately.

For questions, the values given in the table below were obtained.

Mathematics is the science of patterns and relationships. As a theoretical discipline, mathematics explores the possible relationships among abstractions without concern for whether those abstractions have counterparts in the real world. From this thought and Table 2, the degree of agreement about 1st question obtained at "strongly agree" with score 0.69952. That is, PMT strongly believes that the mathematics is made up of abstract concepts. Mathematics, as perceived by society, has a sharp hierarchy of formulas, symbols and processes in general. With the answers they give to the first question, PMT supports the opinion of the society. This answer indicates that PMT accepts the general definition of the nature of Mathematics.

Table 2. Mathematics is a discipline that includes abstract concepts

$S(Q_1)$	$S(Q_2)$	$S(Q_3)$	$S(Q_4)$	$S(Q_5)$	$S(Q_6)$	$S(Q_7)$	Result (max value)
0.64175	0.69952	0.55498	0.46321	0.32562	0.30035	0.35045	strongly agree

Perceptions of the nature and role of mathematics held by society have a major influence on the development of school mathematics curriculum, instruction and research. That is why every part of society has an answer about whether mathematics is a feared field or not. In general, although mathematics is seen as a feared area, PMT do not accept this idea. Then, for the 2nd question in Table 3, the calculation of degree of similarity representing "strongly disagree" with score 0.65963.

Table 3. Mathematics is accepted as a difficult and frightening knowledge domain

$S(Q_1)$	$S(Q_2)$	$S(Q_3)$	$S(Q_4)$	$S(Q_5)$	$S(Q_6)$	$S(Q_7)$	Result (max value)
0.36358	0.36852	0.40085	0.43687	0.64387	0.65963	0.64721	strongly disagree

Mathematics, unlike the other sciences, is not constrained by the real world, but in the long run it contributes to a better understanding of that world. Mathematics is important in real life, because it is used to perform many different daily tasks, such as telling time, reading as odometer, counting change and to make strategic decision in one's work life. Similarly, nearly every profession uses some form of mathematics. Math is a life skill needed by everyone to make even simple choices in regard to insurance rate, investment options and looking at stock market trends and data. Math is necessary for playing board games like chess where one has to make strategic moves. In sports, it is necessary to have knowledge about mathematics branches such as statistics, probability and geometry. In society's view of mathematics, the importance of mathematics in life is always accepted, but the importance of mathematics in education is not understood at the desired level. This thought of society has also affected PMT. Therefore, for the 3rd question in Table 4, the calculation of degree of similarity representing "agree" with score 0.61401. In other words, it seems that PMT are not sufficiently convinced of the importance of mathematics in daily life.

The identity of mathematics is controversial from the ancient world today. What kind of work does mathematics work? What are the features that distinguish mathematics from other intellectual works in terms of the subject and method? Why is mathematics an indispensable tool for phenomenological sciences? What do we understand from mathematical certainty? What does "proof" mean? How can the high educational value of mathematics be explained?

Table 4. For real life, mathematics is very important

$S(Q_1)$	$S(Q_2)$	$S(Q_3)$	$S(Q_4)$	$S(Q_5)$	$S(Q_6)$	$S(Q_7)$	Result (max value)
0.59986	0.61359	0.61401	0.51357	0.55084	0.38691	0.33627	agree

Mathematics is a universal language and it has an important place in other branches of science. It involves information processing, predicting and solving problems using this language. When a problem is encountered, rather than finding out what the answer to the problem is, analyzing the problem with various dimensions is required mathematical thinking. Mathematics cannot be evaluated only by technical and operational aspects. At the same time, the intellectual structure of mathematics should be emphasized. If this is done, PMT will be much more beneficial to their students when they start their professions.

When the answers to this question about the intellectual nature of mathematics are examined, it can be seen that PMT gave "strongly agree" with the score 0.70091, as the answer to 4th question because of the training they received.

Mathematics has been called the language of the universe. Three centuries earlier, Galileo had written, "Nature's great book is written in mathematical language" an opinion that has wholeheartedly been endorsed by physicists of our own time. Scientists and engineers often speak of the elegance of mathematics when describing physical reality. Many scientists have suggested that the universe arises out of pure thought that is couched in the language of abstract mathematics. As can be seen from Table 6, PMT replied that the "agree" with score 0.76908 for 5th question.

Table 5. Reasoning, hypothesizing and logical deduction are the products of the human intelligence and these point out the basic principle of mathematics

$S(Q_1)$	$S(Q_2)$	$S(Q_3)$	$S(Q_4)$	$S(Q_5)$	$S(Q_6)$	$S(Q_7)$	Result (max value)
0.65472	0.70091	0.68694	0.45308	0.41762	0.38947	0.37998	strongly agree

Table 6. Mathematics is the center of physical world or truths of the universe

$S(Q_1)$	$S(Q_2)$	$S(Q_3)$	$S(Q_4)$	$S(Q_5)$	$S(Q_6)$	$S(Q_7)$	Result (max value)
0.65795	0.64948	0.76908	0.46564	0.41035	0.34554	0.34063	agree

One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the students' mathematical competence, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. For this reason, a mathematics teacher should be able to both conceptual thinking and easily solve the examples that are appropriate to the subject. PMT stress the importance of conceptual understanding as a key component of mathematical expertise. Then, for the sixth question, the calculation of degree of similarity representing "strongly agree" with score 0.56589, in Table 7.

Table 7. It is the only way to learn mathematics to understand its conceptual meaning and to perform an application

$S(Q_1)$	$S(Q_2)$	$S(Q_3)$	$S(Q_4)$	$S(Q_5)$	$S(Q_6)$	$S(Q_7)$	Result (max value)
0.52256	0.56589	0.51426	0.44553	0.43962	0.40126	0.38284	strongly agree

Mathematics is a universal language because the principles and foundations of math are the same everywhere around the world. Mathematics is the only language shared by all human beings regardless of culture, religion, or gender. With this universal language, all of us, no matter what our unit of exchange, are likely to arrive at math results the same way. The degree of agreement about last question obtained at "strongly agree" with score 0.56589. Thus, PMT certainly accepts the universality of mathematics with the answers they give to the final question.

Table 8. Mathematics is the biggest shared value of human brain. So, universality is the power of mathematics

$S(Q_1)$	$S(Q_2)$	$S(Q_3)$	$S(Q_4)$	$S(Q_5)$	$S(Q_6)$	$S(Q_7)$	Result (max value)
0.63386	0.72034	0.65998	0.49621	0.43123	0.36951	0.36362	strongly agree

Mathematics relies on both logic and creativity, and it is pursued both for a variety of practical purposes and for its intrinsic interest. For some people, and not only professional mathematicians, the essence of mathematics lies in its beauty and its intellectual challenge. For others, including many scientists and engineers, the chief value of mathematics is how it applies to their own work. Because mathematics plays such a central role in modern culture, some basic understanding of the nature of mathematics is requisite for scientific literacy. To achieve this, students need to perceive mathematics as part of the scientific endeavor, comprehend the nature of mathematical thinking, and become familiar with key mathematical ideas and skills. (The Nature of Mathematics: Science for all Americans [32]).

Research on teachers' beliefs is rooted in the idea that beliefs structure people's interactions with the world - and thus teachers' interactions with their students in the school context- and consequently influence their perceptions, goals, and behaviors (Voss et al. [27]). According to Schoenfeld [19], mathematics teachers' beliefs can be thought of as an individual's perspective on how one engages in mathematical tasks and pedagogical practices. A growing body of literature shows that mathematics teachers' beliefs affect their classroom practices (Pajares [17], Thompson [25]).

Beliefs of PMT are critical foci for teacher education because these beliefs may influence how they participate in and understand mathematics. Foss and Kleinsasser [6] mentioned that many researchers advocate an increase in the use of qualitative methods and suggest future studies include emphasis on how PMT beliefs align with various conceptions of teaching and learning.

4. Conclusions

There is no definition of belief that everyone agrees in the field of education. However, widespread opinion is that beliefs are shaped by personal experiences, culture and education. Beliefs help us in understanding teachers teaching and they are constructed through teachers' experiences (Pajares [17], Kayan et al. [12]).

The theory of fuzzy sets now encompasses a well-organized corpus of basic notions including (and not restricted to) aggregation operations, a generalized theory of relations, specific measures of information content, a calculus of fuzzy numbers.

It is difficult to explain uncertain situations such as beliefs or belief systems by numerical evaluations or to produce strict ideas with precise computation. In this study, beliefs were measured with the fuzzy conjoint model and quantitative results were obtained, even if it is difficult to measure uncertain situations such as beliefs. In this study, we presented an example of research work using the fuzzy theory which has been used to model a variety of situations in social sciences. The beliefs of the PMT were transformed into the values of the membership function in the fuzzy set theory. Then, degree of similarity was calculated with the given model. Thus, the beliefs of the PMT about the nature of mathematics and the learning of mathematics were determined.

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