https://jksus.org

Journal of King Saud University – Science

Article in Press



Research Article

Optimization of the Sierpinski carpet fractal antenna with a multi-repetition factor for multi-band wireless applications utilizing a variant strength Pareto evolutionary algorithm

Ahmet Uluslu[a,*]

[a]Department of Electronics and Automation, İstanbul University-Cerrahpaşa, Istanbul, 34500, Turkey

ARTICLE INFO

Keywords:
Fractal antenna
MOPs
mSPEA2
Multi-band
Optimal characterization
Optimization techniquesSierpinski carpet
Strength pareto evolutionary algorithm

ABSTRACT

Designing a multiband, compact, and high-performance antenna is considered a multi-objective and high-dimensional optimization problem, as it requires the simultaneous satisfaction of multiple performance criteria (such as gain and return loss). In this study, a six-variable design problem is formulated for a Sierpinski carpet fractal patch antenna (SCFPA) operating in the 3.5 GHz, 4.1 GHz, 5.1 GHz, 5.8 GHz, and 7.7 GHz frequency bands. Within this scope, a strength Pareto evolutionary algorithm 2 (SPEA2) based approach is developed, and the algorithm is extended with two original variants: (i) an improved archive update mechanism (mSPEA2) to increase solution diversity, and (ii) an adapted crossover/selection strategy (cSPEA2) to improve convergence performance. The proposed method aims to both iteratively incorporate antenna geometry into the optimization and simultaneously achieve multiband performance objectives. According to the results obtained, the proposed variant algorithms perform better compared to the standard SPEA2. Specifically, with the mSPEA2 algorithm, the following values were obtained: 1.32 dB gain and −16.38 dB S_{11} at 3.5 GHz, 1.97 dB gain and −25.58 dB S_{11} at 4.1 GHz, 0.83 dB gain and −13.11 dB S_{11} at 5.1 GHz, 0.60 dB gain and −23.67 dB S_{11} at 5.8 GHz, and 5.91 dB gain and −13.84 dB S_{11} at 7.7 GHz. These results indicate improvements, particularly in terms of return loss and multiband operation. The proposed antenna structure is as follows: The proposed SPEA2 variants are designed to be compatible with WLAN/Wi-Fi (IEEE 802.11a/ac) and 5.8 GHz ISM applications in the 5–6 GHz band, WiMAX applications in the 3.5 GHz and 4.1 GHz bands, and short-range wireless communication systems such as Bluetooth around 5 GHz. Consequently, the proposed SPEA2 variants offer a flexible and effective optimization approach for multi-band fractal antenna design problems and can be easily adapted to different microwave optimization problems.

1. Introduction

In modern communication systems, antennas are critical components that provide the link between the receiver and transmitter. Research conducted particularly between 2021 and 2026 (Çolak *et al.*, 2026; Gocen *et al.*, 2024; Mahouti *et al.*, 2026; Niyaz *et al.*, 2024; Palandöken *et al.*, 2024; Piltan *et al.*, 2024; Yurt *et al.*, 2024), emphasizes the importance of antenna design in wireless communication simultaneously meeting multiple criteria such as high gain, directivity, low reflection coefficient, multiband capability, small size, low cost, and ease of manufacturing (Balanis, 2016; Das *et al.*, 2023; Kumari and Hossain, 2024; Qiong *et al.*, 2024; Yi *et al.*, 2023). Microstrip patch antennas are widely used in current wireless systems because they meet many of these characteristics. These antennas offer advantages in terms of size, weight, and ease of assembly in applications such as aircraft, missiles, satellites, and mobile devices (Balanis, 2016). However, since microstrip antennas tend to operate in a single band, performance limitations are observed in multiband or broadband applications (Yi *et al.*, 2023).

The history of microstrip patch antennas begins with their discovery in 1953 and patenting in 1955 (Balanis, 2016). They became popular in the 1970s due to the need for small-profile antennas in radar systems, satellite, and mobile communications (Balanis, 2016). This historical reference is supported by many sources (Chen *et al.*, 2026; Elahi *et al.*, 2026; Liu *et al.*, 2026). Their main advantages are small profile, robustness, low cost, and easy integration, while their disadvantages are narrow bandwidth, limited power, and low efficiency (Hall, 2009; Long and O'Connor, 2016). To overcome these disadvantages, antennas with fractal geometry have been designed. The word fractal means broken pieces or irregular (Das *et al.*, 2023). It is of Latin origin and is derived from the word 'fractus'. It was first defined by Benoit Mandelbrot (Ge *et al.*, 2024). Fractal antennas are inspired by nature and have two basic properties: self-similarity and gap-filling (Sinha *et al.*, 2024). Self-similarity means that the main part of the antenna can be divided into smaller parts, which is the smallest part of the main part (Singh *et al.*, 2013). Another feature is gap filling, which reduces the main size of the antenna, making it longer in electrical dimension (Shrestha *et al.*, 2013). These structures are also called multi-level gap filling curves (Babu *et*

***Corresponding author:**
E-mail address: auluslu@iuc.edu.tr (A. Uluslu)

Received: 05 October, 2025 **Accepted:** 10 April, 2026 **Epub Ahead of Print:** 18 June, 2026 **Published:** ***

DOI: 10.25259/JKSUS_1560_2025

This is an open-access article distributed under the terms of the Creative Commons Attribution-Non Commercial-Share Alike 4.0 License, which allows others to remix, transform, and build upon the work non-commercially, as long as the author is credited and the new creations are licensed under the identical terms.

© 2026 Journal of King Saud University – Science - Published by Scientific Scholar

al., 2024; Kattimani and R.Patil, 2023; Mahatmanto et al., 2023; Xu et al., 2024; Zheng and Chen, 2023). Thus, the biggest disadvantage of microstrip antennas is overcome, and multi-band or wideband antennas are made possible. Fractal antenna designs are frequently encountered in simulation software (Yu et al., 2010). There are many fractal geometric antenna structures under the Sierpinski model, such as the Sierpinski carpet, Koch curve, and Minkowski Hilbert curve (Banerjee et al., 2018; K and Santhanam, 2024; Koniammal et al., 2023; Mainkar, 2023). It provides a connection in wireless communication, especially in the wireless local area network (WLAN) and worldwide interoperability for microwave access (WiMAX) bands. It is frequently preferred in mobile devices in the WLAN and WiMAX bands (Ramkumar et al., 2021).

Referring to the studies in the literature, one of the first antenna designs based on fractal geometry, the mod Sierpinski fractal antennas, is derived from Pascal's triangle and exhibits log-periodic behavior as a result of self-similarity properties. Mod Sierpinski fractal antennas constitute a generalization of the classical Sierpinski antenna (Romeu and Soler, 2001). There are studies on the design of 2 and 3-band fractal antennas in different ways inspired by nature (Chaouche et al., 2019; Islam et al., 2016; Lizzi and Massa, 2011). There are also many designs that have been replicated by taking advantage of the self-similarity property (Rao and Dinesh Kumar V., 2014). There is a study that shows that the narrow band feature, which is a disadvantage of microstrip patch antennas, has been overcome (Kirubavathy and Ramprakash, 2017). In one study, the parameters of a rectangular patch antenna were optimized using a genetic algorithm, and performance analysis was performed using CST software. In this study, the genetic algorithm was implemented in the CST environment using MATLAB (Lamsalli et al., 2016). In another study, the electromagnetic simulation (EMS) HFSS environment was automated using MATLAB code, and the substrate integrated waveguide (SIW) cavity slot antenna structure was designed and optimized using the particle swarm optimization (PSO) algorithm (Dahbi et al., 2022). The maximum gain value of 8.8 dBi obtained in this study offers a significant improvement compared to conventional planar antennas, and a good impedance match is achieved with a low S_{11} parameter (Dahbi et al., 2022). However, such studies generally focus on a single performance metric (e.g., gain or reflection coefficient) and are limited in addressing multiband operation requirements simultaneously.

Similarly, although different antenna designs aimed at achieving high gain have been proposed in the literature (Uduwawala et al., 2019), most of these studies either focus on a limited number of bands or increase the complexity of the antenna geometry, leading to production difficulties in practical applications. When the proposed designs for WLAN and WiMAX bands are examined (Aggarwal and Kartikeyan, 2010; Azaro et al., 2009; Ghatak et al., 2008; Yadav et al., 2018), it is generally seen that solutions optimized for specific frequency bands are offered, but there are shortcomings in addressing the requirements of wide or multi-band performance and compact structure together. Studies on fractal antenna design optimization are also present in the literature (Aggarwal and Kartikeyan, 2010; Koziel et al., 2017; Nguyen Duc Hieu et al., 2010; Roy et al., 2016; Sankhe and Khot, 2018). However, in most of these studies, it is observed that either the optimization algorithms used are limited to a single structure or the variable geometric parameters of the antenna are not fully integrated into the optimization process. In addition, it is noteworthy that multi-objective optimization approaches are not used comprehensively enough, especially in addressing performance criteria such as multi-band operation, gain and reflection coefficient simultaneously. Optimization and design studies in different disciplines have been guiding in terms of developing solution strategies for multivariate and multi-objective problems (Agarwal et al., 2025, 2003, 2001; A. Mishra et al., 2004; A Mishra et al., 2004; Mishra et al., 2003, 2002; Muwal et al., 2025; Shankar et al., 2020, 2018, 2017, 2014; Singh et al., 2020, 2019, 2018). These studies include research carried out in the fields of polymer science and chemistry using optimized parameters and iterative modeling techniques; similar methods can also be applied to the optimization of fractal antenna design.

In this context, the literature lacks a holistic approach that simultaneously addresses multiband, compact, and high-performance antenna design and incorporates iterative changes in antenna geometry into the optimization process. This study aims to fill this research gap by combining fractal antenna design with multi-objective optimization algorithms to simultaneously provide the advantages of both size reduction and bandwidth increase. Thus, a multiband fractal antenna has been designed that improves both gain and S_{11} parameters and can be used in modern wireless communication systems (WLAN, WiMAX, Wi-Fi, Bluetooth).

In this study, a multiband antenna structure operating at 3.5 GHz, 4.1 GHz, 5.1 GHz, 5.8 GHz, and 7.7 GHz frequencies was developed by combining fractal antenna design with multi-objective optimization approaches. The large number of antenna design parameters and the requirement for five-band operation make the optimization process quite complex. To overcome this challenge, an optimization approach based on the strength Pareto evolutionary algorithm 2 (SPEA2), adapted from mathematical models and supported by a problem-specific objective function, was used.

The scientific contributions of this study are summarized below:

- The standard SPEA2 algorithm has been adapted to the multiband fractal antenna design problem and made problem-specific.
- In addition to the standard SPEA2, two different variant algorithms have been developed to increase solution diversity and convergence performance.
- Variable parameters of the antenna geometry have been effectively included in the optimization process, adopting an iterative design approach.
- A multiband antenna design covering the frequencies 3.5 GHz, 4.1 GHz, 5.1 GHz, 5.8 GHz, and 7.7 GHz has been realized, and performance improvements have been achieved.
- The proposed method has been validated in a simulation environment, and the results obtained have been analyzed in terms of different performance criteria (gain, S_{11}, etc.).

In these aspects, the study offers original contributions to the literature in terms of both algorithmic development and antenna performance improvement.

In order to validate the results, all these optimization processes are considered as a whole, and the design of the most successful result is also displayed in another program, the 3D EM simulation program CST Microwave Studio environment. The study has several innovations with variant optimization algorithms supported by original objective functions. Considering similar studies in the literature, the inclusion of the physical structure that changes with the number of bands and the number of antenna iterations in the optimization and the fact that all this is done with 2 different variant new algorithms takes the study very far (Aggarwal and Kartikeyan, 2010; Koziel et al., 2017; Nguyen Duc Hieu et al., 2010; Roy et al., 2016; Sankhe and Khot, 2018).

This study is broken down into the following chapters, which are structured as follows: In Chapter II, under the methodology title, multi-objective optimization is briefly mentioned, and information is given about the objective and cost functions used together with the existing SPEA2 and variant optimization. Antenna design considerations are discussed in Chapter III. The results of the study were analyzed in Chapter IV. Chapter V contains a discussion of what has been done. The article ends in Chapter VI.

2. Methods

Classical optimization and search algorithms cannot provide adequate performance for nonlinear, complex, and dynamic large-scale problems with missing information (Akyol and Alatas, 2017; Alatas and Bingol, 2020; Hillier and Lieberman, 2005). In particular, in multivariate antenna design problems, multiple performance criteria (e.g., gain, reflection coefficient, and bandwidth) need to be optimized simultaneously. Such problems are defined as multi-objective optimization problems and involve conflicting objectives that cannot be expressed by a single objective function. Multi-objective antenna optimization problems have multiple alternative best solutions in the solution space. This set of solutions is called the Pareto optimal set

and provides an optimal equilibrium where no solution is superior to another. In antenna design, the objective vector contains the performance criteria to be optimized and is expressed as follows:

$$\text{Minimize } \vec{y} = F(\vec{x}) = \left[f_1\vec{x}, f_2\vec{x}, \ldots, f_N\vec{x} \right]^T$$

$$\text{Subject to } g_j(\vec{x}) \le 0, \quad j = 1,2,\ldots,M$$

In Here:

$\vec{x} = \left[x_1, x_2, \ldots, x_p \right]^T \epsilon\, \Omega$ represents the decision variables and antenna design parameters (e.g., patch sizes, gap ratios, fractal iteration parameters).

$g_j(\vec{x})$ represents the constraints in antenna design (e.g., physical size, manufacturing limitations).

y is the objective vector and contains the performance metrics to be optimized (gain, S_{11}, bandwidth, etc.).

Ω is the feasible objective space that satisfies all constraints.

Thanks to this structure, multi-objective optimization enables the simultaneous improvement of both geometric parameters and performance criteria in antenna design. Solutions obtained with the Pareto-based approach offer various alternatives without being reduced to a single criterion. In this study, the SPEA2, whose effectiveness has been proven in the literature, was used, and two problem-specific variants were developed to effectively include the antenna geometry in the optimization process (Yang *et al.*, 2025). Thanks to these methods, both the variety of solutions was increased, and the convergence performance was improved in multi-band fractal antenna design.

2.1 The strength Pareto evolutionary algorithm 2

To implement SPEA2, an initial population and an archive are first determined. At the beginning of the process, the fitness value of the population is computed, and power values are assigned to each individual that is a part of the population (Zitzler *et al.*, 2001).

For each individual *i* in the archive, a power value *S(i)* is assigned [0;1], which also indicates its fitness value *F(i)*. *S(i)* is the number of individuals in the population equal to or dominated by *i*, depending on the objective value *j*, divided by 1 more than the population size. In the population, the fitness value *F(j)* of individual *j* is equal to or dominating *j*, *i* is equal to 1 more than the sum of the *S(i)* values in all archive members.

The steps of the SPEA2 algorithm are defined as follows.

Algorithm 1. Pseudocode of SPEA2. Input parameters; population size (*N*), archive size (*NA*), maximum number of generations (*TG*). Output parameters; non-dominated archive (*A*).

Step 1: Create an initial population P_0 and an empty archive A_0. Set $t = 0$.

Step 2: Calculate the fitness values for each individual in P_t and A_t.

Step 3: Copy all non-dominated individuals in P_t and A_t to A_{t+1}.

Step 4: If $|A_{t+1}| >$ N A, apply the truncation operator based on density estimation to reduce the archive size to *NA*.

Step 5: If $|A_{t+1}| <$ N A, fill A_{t+1} with dominated individuals from P_t U A_t sorted according to their fitness values.

Step 6: If ($t = TG$) or if the agreed stopping criterion is met, set $A = A_{t+1}$ and stop.

Step 7: Apply selection at A_{t+1} to fill the match pool.

Step 8: Apply crossover and mutation to the matching pool and update A_{t+1}. Set $t = t+1$ and go to Step 2.

In most multi-objective evolutionary optimization algorithms, three basic processes are crucial to optimization success. These are briefly described below.

Availability sharing: It advocates that individuals with a certain suitability should have access to common resources. The more individuals in the same neighborhood, the lower the fitness value will be.

Environmental selection: An archive is kept apart from the population. This archive contains non-dominated solutions in all populations so far. It is possible that the person will be deleted from the archive if another solution is discovered that is more effective than the individual, or if the maximum size of the archive is surpassed. Often, the only way for an individual to survive is by archival duplication over several generations. In addition, the same individual can only be reproduced by chance. The aim here is to prevent the loss of existing non-dominated solutions due to random factors.

Mapping selection: Each generation undergoes a two-stage evaluation of its individuals. Initially, all persons are evaluated based on the Pareto dominance relationship. Basically, how many individuals an individual dominates in the pool of individuals determines that individual's rank. Then these rows are evaluated according to the density in the pool. Various techniques can be used for this.

2.2 Variants of SPEA2

By applying two different basic approaches to the three basic processes mentioned above, 2 different variants were derived.

These are, in order;

- It is the integration of SPEA2's compliance calculation to step 2 in algorithm 1. This variant has been named cSPEA2.
- Integration of SPEA2's mapping selection into step 7 in algorithm 1. This variant is named mSPEA2.

Variant 1 (cSPEA2): Availability calculation is a critical process in comparing and ranking individuals. Obtained power values are accepted as fitness values and directly affect the selection process. All steps of SPEA2 are implemented as determined, and fitness functions are assigned to individuals. When individuals with the same fitness value are evaluated in binary tournament selection, a second fuzzy fitness value is assigned to separate them. Individuals are now parsed according to these fuzzy fitness values and added to the matching pool. Transactions are continued without making any changes in other processes of SPEA2. This approach provides an effective solution, especially in the elimination of individuals with similar and the same suitability as the search progresses. It also provides the benefit from the experiences of decision makers.

Algorithm 2. Pseudocode of fitness calculation integration.

Step 1: Randomly select two individuals from the population and archive set: *i* and *j*.

Step 2: Compare the fitness of individuals *i* and *j*. If it's the same, go to Step 3, otherwise exit.

Step 3: Calculate the fuzzy fitness values of *i* and *j* according to linguistic variables and rules.

Step 4: Apply the selection process using the fuzzy fitness values obtained.

Variant 2 (mSPEA2): The SPEA2 algorithm compares the fitness levels of two randomly chosen people using a double tournament selection method. It is important that the selected individuals have a certain distance to increase diversity in the population and to minimize premature convergence. For this reason, fuzzy logic is used to keep the two individuals considered at a certain distance, considering all purposes. In the selection phase, the neighborhood level of two individuals is determined by fuzzy logic rules (Almadi *et al.*, 2022). If this neighborhood level is not within the desired fuzzy limits, a different individual is taken, and the process continues. In some cases, another individual with the appropriate distance may not be available. In these cases, the most distant individual is selected appropriately or mutated and processed. The biggest advantage of this approach is that it increases the diversity in the population and offers a widespread distribution.

Algorithm 3. Pseudocode of the mapping selection.

Step 1: Randomly select two individuals from the population and archive set: *i* and *j*.

Step 2: Determine the neighborhood level of individuals *i* and *j* within the framework of fuzzy rules.

Step 3: If *i* and *j* have the desired neighborhood level (distance is sufficient), apply the selection operation.

Step 4: Otherwise, randomly pick a new individual from the cluster and go to Step 2.

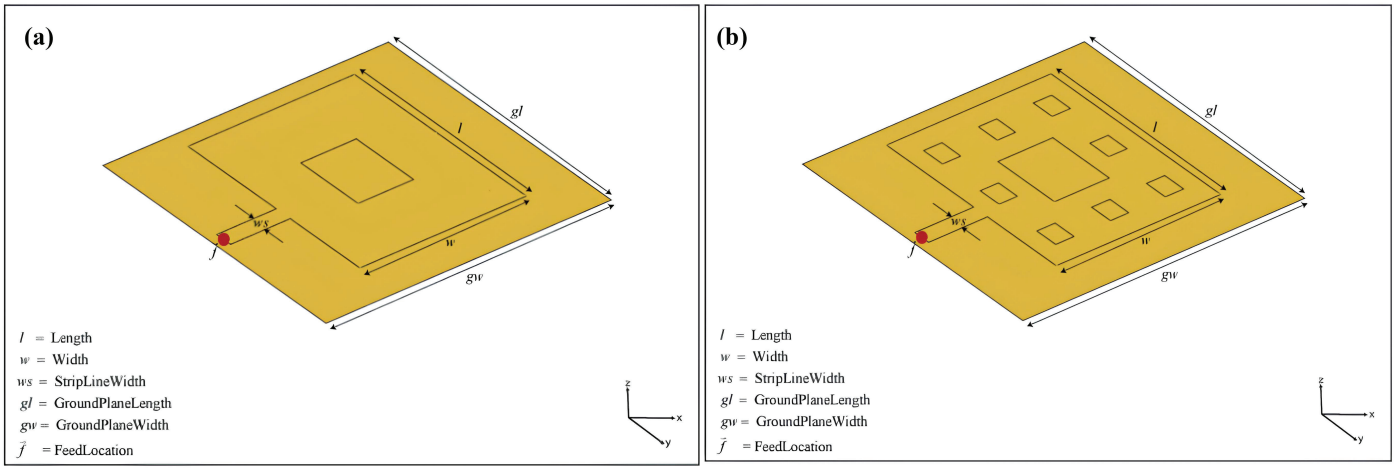


Fig. 1. Physical parameters according to the number of iterations performed in the fractal antenna. (a) 1, (b) 2.

The original algorithm was enhanced by the addition of two innovations, which were designated as cSPEA2 and mSPEA2, respectively, representing variants of the original SPEA2. This study incorporates two variants alongside the original SPEA2.

2.3 Objective and cost functions

In the study, based on a previous study, the adapted objective function pair will be defined, inspired by mathematical methods (Chapra and Canale, 2021). This objective function pair, which will be used in the optimization algorithms SPEA2, mSPEA2, and cSPEA2, is based on a nonlinear mathematical method and is named as the exponential model. The decibel value of the reflection coefficient, or return loss, is equivalent to the negative decibel value of the S_{11} parameter, commonly utilized in simulations. Typically, simulation tools compute S_{11} and present it in decibels (dB). The directivity of an antenna is the ratio of the power density in the direction of maximum radiation to the power density produced by an isotropic antenna with equivalent power at the same distance.

Two measurement functions, the decision variables S_{11} and directivity (D), defined above, were used to create the objective function pair. Here, the objective of the optimization algorithm is to make S_{11} as low as possible (max. -10dB) and to have high directivity. In addition, since the importance of the decision variables may change according to the problem, the weighting coefficients ($wc_{1\text{-}2}$) are added to the objective function pair.

Exponential model: Within the realm of mathematics, the model that is referred to as an exponential function or exponential function is utilized rather frequently. The broad definition of it is a^x. But here, e is often used instead of a. The value that is desired to be low in the function is given with a positive (+) coefficient at the top, and the value that is desired to be high is given with a negative (-) coefficient at the top. Weighting coefficient values ($wc_{1\text{-}2}$) are added to the expressions as multipliers (Uluslu, 2023). Thus, the objective function pair adapted from the exponential model is defined as follows Eq. (1-3):

$$OF_1 = \sum_{i=1}^{N} WC_1 * e^{-D_i} \quad (1)$$

$$OF_2 = \sum_{i=1}^{N} WC_2 * e^{S11_i} \quad (2)$$

where i=*1, 2.,5,* each represents a resonance frequency.

The cost function to be used to determine the optimum result; It is obtained by adding the determined objective function pairs with each other and is defined as follows.

$$\text{Cost} = OF_1 + OF_2 \quad (3)$$

3. Design Process of Antenna

In recent years, two or three band antennas are needed in wireless communication systems. For example, there are studies for WiMAX band applications (Aggarwal and Kartikeyan, 2010; Azaro *et al.*, 2009; Ghatak *et al.*, 2008; Yadav *et al.*, 2018). Here, a multi-band antenna model with five resonance frequencies, 3.5 GHz, 4.1 GHz, 5.1 GHz, 5.8 GHz, and 7.7 GHz, with a Sierpinski carpet fractal antenna (SCFPA) structure, will be designed to meet these similar needs. The Sierpinski fractal antenna has a basic structure consisting of a microstrip patch (Gupta, 2000). Thanks to a few iterations in the model, they take on a multi-band characteristic. SCFPA is a repeating pattern in which a square patch is removed. All fractal antennas qualify as printed structures etched onto a dielectric substrate. The configuration and repetition of the square patch are seen in Fig. 1. The substrate thickness was selected as a variable. Fig. 1 illustrates the antenna design schematic using iterations one and two. The ranges of design parameter values indicated in the picture are elaborated in Table 1. All design steps are delineated in Algorithm 4. Of the total 8 design parameters, 6 are variable, and 2 are fixed; variable parameters were determined to increase design flexibility and optimization success, while fixed parameters were kept constant to maintain basic structural and performance constraints. In the optimization processes performed with MATLAB, PEC was selected as the conductor and FR4 (ε_r = 4.4, loss tangent tanδ = 0.02) as the substrate. Just like in microstrip patch antennas,

Table 1.
Design parameters of the proposed antenna.

Parameter	Definition	Center value	Value range
NumIterations	Number of iterations performed on the fractal antenna	-	1 or 2
Length	Length of fractal carpet along the x-axis	27.9 (mm)	±25%
Width	Width of fractal carpet along the y-axis	37.25 (mm)	±25%
Height	Height of the fractal carpet above ground	1.59 (mm)	±25%
StripLineWidth	Width of the feeding strip line	3 (mm)	±25%
Substrate	Type of dielectric material	-	FR4
GroundPlaneLength	Length of ground plane	48 (mm)	±25%
GroundPlaneWidth	Width of ground plane	57 (mm)	±25%
FeedOffset	Signed distance of feed from origin	-	(-Length/2, Length/2)
Conductor	Type of metal material	-	PEC

although there are empirical equations with many methods in fractal antenna designs, both the technique is old, and there is complexity and inconsistency (Milovanovic *et al.*, 2006; Romeu and Soler, 2001). Despite this complexity, electromagnetic simulation has become extremely common in recent years. This investigation makes use of the antenna toolbox that is included in MATLAB 2024B for the purpose of optimizing processes and simulating antennas. A machine with a 13900K Intel Core i9 CPU that was working at 5.80 GHz, a graphics card that had 16 GB of memory, and 32 GB of random-access memory was used to carry out all of these tasks. In addition, the results were validated in the CST Microwave studio environment, which makes use of the 3D electromagnetic simulation tool.

Algorithm 4. Fractal antenna design process.

Step 1: Along with the fixed parameters, the center values and value ranges of the variable parameters are entered.

Step 2: Optimization is started with randomly assigned initial values.

Step 3: S_{11} and directivity (D) are obtained by performing a separate simulation with the MATLAB program for each input parameter group.

Step 4: The algorithm is run 10 times separately in order to minimize both of the determined objective functions. (The internal processes of the algorithms are detailed in Chapter II)

Step 5: Among the results obtained, the result with the minimum cost value is declared the most successful.

Step 6: The most successful result obtained is verified by the CST Microwave Studio environment.

In the next chapter, a comprehensive illustration of the working example will be shown on the optimization and simulation verification of the fractal antenna with the design parameters specified in this section for the frequency bands 3.5 GHz, 4.1 GHz, 5.1 GHz, 5.8 GHz, and 7.7 GHz.

4. Study Case

In the study part, the first thing that will be done is to identify the ideal parameters for the method that will be utilized in antenna optimization within the parameters that have been specified for the design. The original goal function that was developed from the mathematical model that was created by these particular characteristics will be utilized in the subsequent parts. During the subsequent phase of the investigation, a comparison will be made between the performance of the alternative algorithms that are generated from SPEA2 and their counterparts. This is due to the fact that the priority of the choice factors that are incorporated into the measurement functions may be subject to change. Additionally, several weighting coefficients ($wc_{1\text{-}2}$) will be tested. After all of these steps have been completed, it will be completed with a 3D electromagnetic simulation in order to get the best possible outcome. Since the methods used in this study are stochastic, different results may be obtained in each run. Therefore, before the optimization processes, the random number generator was fixed for each run using the `rng(n)` command in MATLAB (n = 1, 2, …, 10). Thus, each optimization was performed with different predetermined random starting conditions, and the best performance obtained from 10 different runs of the code was considered. All processes were applied sequentially under equal conditions.

4.1 Performance comparison with different algorithms

The performance of the proposed SPEA2 algorithm was compared with PSO and differential evolution algorithms (DEA), which are widely used in the literature for multi-objective optimization problems. Performance evaluation was carried out using the *cost* (3) function, which was determined using the pair of objective functions OF_1-OF_2 (1,2) detailed in previous sections. The maximum number of iterations was set to 30 and the population size to N = 50 for all algorithms. Fig. 2 shows the best performance iterations of PSO, DEA, and SPEA2 algorithms, along with typical cost and function evaluation number (FEN) variations. For each algorithm, the cost value was considered as the lowest value selected from 10 different runs. In addition, the minimum, maximum, mean, and standard deviation values obtained from 10 different runs are presented numerically in Table 2. The results show that the proposed SPEA2 algorithm exhibits high performance for this problem and provides an advantage compared to the PSO and DEA algorithms.

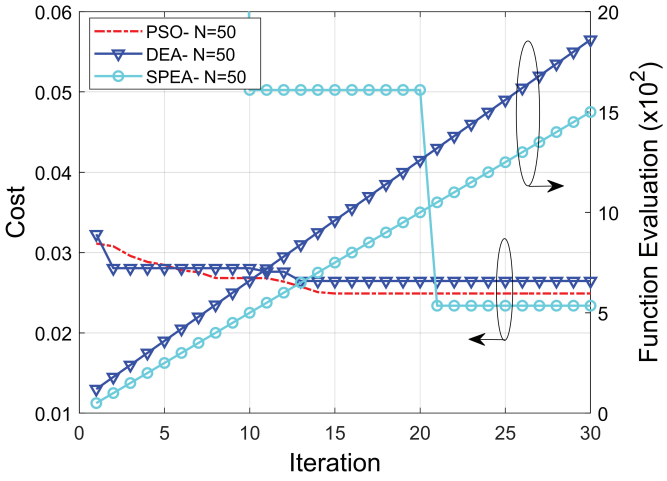


Fig. 2. Comparison of cost-FEN/iteration variations of the best performance selected from 10 runs for PSO, DEA, and SPEA2 performance comparison.

Table 2.
Algorithm performance evaluations for the results in Fig. 2.

Algorithm		Minimum	Maximum	Mean	Standard deviation
PSO	Cost	0.0249	0.0926	0.0311	0.0019
	FEN	800	100	1550	-
DEA	Cost	0.0265	0.0502	0.0323	0.0012
	FEN	700	100	1550	-
SPEA2	Cost	0.0234	0.7852	0.1478	0.2058
	FEN	1050	50	1500	-

4.2 Optimal parameter selection for optimization

Population diversity in evolutionary algorithms is very important for the efficiency of Pareto optimal results. In addition, the preferred algorithm is a population-based algorithm. The goal of most evolutionary algorithms is to eventually arrive at a single solution, which often prevents us from finding the correct optimal results. For all reasons, the choice of population size is important. Choosing a large population size will not only prolong the solution process but also waste resources. On the contrary, if it is chosen small, it will result in the local optimum being stuck and the global optimum not being found (Reeves and Rowe, 2002). For this reason, the process of determining the optimum population parameter for 4 different values was performed. Comparisons of performance were carried out over the *cost* (3) function, which was established by employing the objective function pair OF_1-OF_2 (1,2) that was described in the parts that came before this in the article. Since the optimization algorithms are based on randomly started conditions and processes, the algorithm code is run 10 times for a fixed maximum iteration and different population parameters, as in other optimization problems. The findings that were discovered are presented in Fig. 3 as cost-FEN/iteration variations of the highest performance that was chosen from ten separate experiments. In addition, the differences in cost and FEN that are shown in Fig. 3 are presented in the form of a numerical table in Table 3. Among the findings that were collected for this investigation, it was discovered that the population (N) value, which is the one that yields the outcome with the most optimal cost, is 100. The value of the optimization time (N) was determined to be between 100 and 192 minutes.

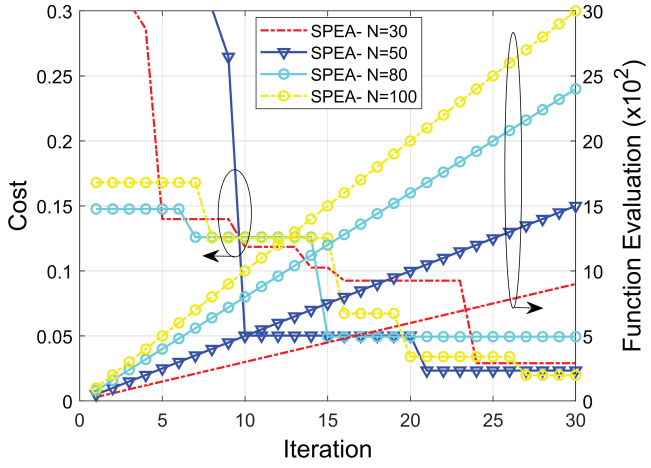


Fig. 3. Comparison of cost-FEN/iteration variations of the best performance selected among 10 runs for population (*N*) parameter selection.

Table 3.
Performance evaluations of the algorithm by population (N) parameter (maximum iteration = 30).

Population		Minimum	Maximum	Mean	Standard deviation
30	Cost	0.0291	0.6301	0.1284	0.1206
	FEN	720	30	900	-
50	Cost	0.0234	0.7852	0.1478	0.2058
	FEN	1050	50	1500	-
80	Cost	0.0495	0.1476	0.0895	0.0442
	FEN	1200	80	2400	-
100	Cost	0.0198	0.1680	0.0922	0.0576
	FEN	2700	100	3000	-

4.3 Performances of SPEA variants

Since the SPEA2 algorithm used in the study is a very rooted algorithm, it is included in the variant models derived from the continuation of the study. In this context, SPEA2, cSPEA2, and mSPEA2 performances were compared using the optimum population value determined in the previous section and equal weighting coefficients (w_i=0.5). The values obtained as a result of the optimization are given numerically in Table 4 Also, in Fig. 4, the return loss (S_{11}) variation in the 3-8 GHz band range is given as a typical magnitude-frequency. When the results are examined in general, it is seen that the S_{11} parameter is below the acceptable level of -10 dB. The gain was found to be higher

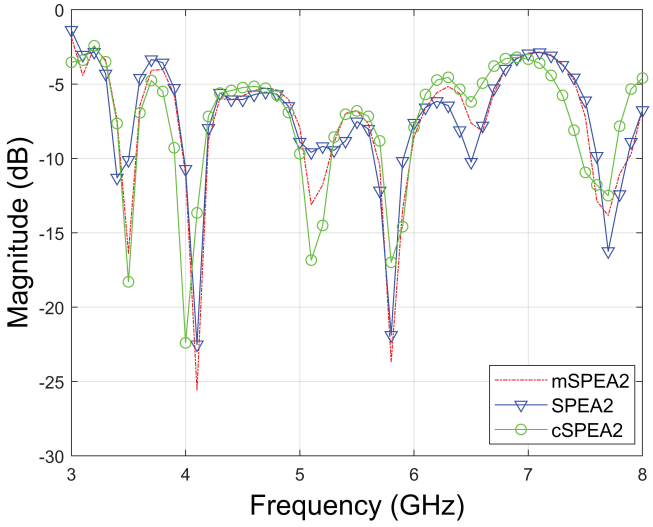


Fig. 4. Return loss of the fractal antenna obtained using MATLAB.

for high frequency bands. As can be seen from the results, mSPEA2 is the most successful variant algorithm model.

4.4 Effect of weight coefficients on optimization

Since the importance of the decision variables in the optimization process may vary according to the problem, the objective function pair derived from the mathematical models was designed with a variable weights. w_i creates weights and is generally taken as $\sum w_i = 1$. One of the disadvantages of this method is that it misses optimal solutions for some purposes due to weighting. Therefore, not choosing these parameters properly may result in not obtaining good solutions. This situation can be adjusted with the weight coefficients ($w_{1\text{-}2}$) added to the objective function pair. Accordingly, 3 different weight coefficients ($w_{1\text{-}2}$=0.3-0.7), ($w_{1\text{-}2}$=0.5-0.5), and ($w_{1\text{-}2}$=0.7-0.3) were studied, and the results of the optimum cost value were given in Table 5 Also, in Fig. 5, the return loss (S_{11}) variation in the 3-8 GHz band range is given as a typical magnitude-frequency. The combination of $w_{1\text{-}2}$ = 0.5 provides a balanced optimization by giving equal weight to both objective functions. Therefore, the best performance in this study was obtained with this weighting combination.

4.5 Radiation pattern of the antenna

Until this part of the study, the design parameters for the most optimum antenna model were found on the MATLAB program by using

Table 4.
Performance results of algorithms by variants (wi=0.5).

	3.5 GHz		4.1 GHz		5.1 GHz		5.8 GHz		7.7 GHz	
	Gain (dB)	S_{11}(dB)	Gain (dB)	S_{11}(dB)	Gain (dB)	S_{11}(dB)	Gain (dB)	S_{11}(dB)	Gain (dB)	S_{11}(dB)
mSPEA2	1.32	-16.38	1.97	-25.58	0.83	-13.11	0.60	-23.67	5.91	-13.84
SPEA2	0.88	-10.11	1.98	-22.51	0.08	-9.59	1.82	-21.88	6.09	-16.24
cSPEA2	1.73	-18.30	1.76	-13.66	0.63	-16.84	-0.55	-16.98	5.47	-12.50

Table 5.
Performance results of the algorithm by different weighting coefficients (mSPEA2).

$w_{1\text{-}2}$	3.5 GHz		4.1 GHz		5.1 GHz		5.8 GHz		7.7 GHz	
	Gain (dB)	S_{11}(dB)	Gain (dB)	S_{11}(dB)	Gain (dB)	S_{11}(dB)	Gain (dB)	S_{11}(dB)	Gain (dB)	S_{11}(dB)
0.3-0.7	1.78	-10.72	2.11	-8.41	-1.71	-2.58	-1.45	-3.75	4.17	-12.86
0.5-0.5	1.32	-16.38	1.97	-25.58	0.83	-13.11	0.60	-23.67	5.91	-13.84
0.7-0.3	2.03	-10.40	1.87	-7.01	-3.21	-5.02	-3.02	-5.51	3.27	-13.64

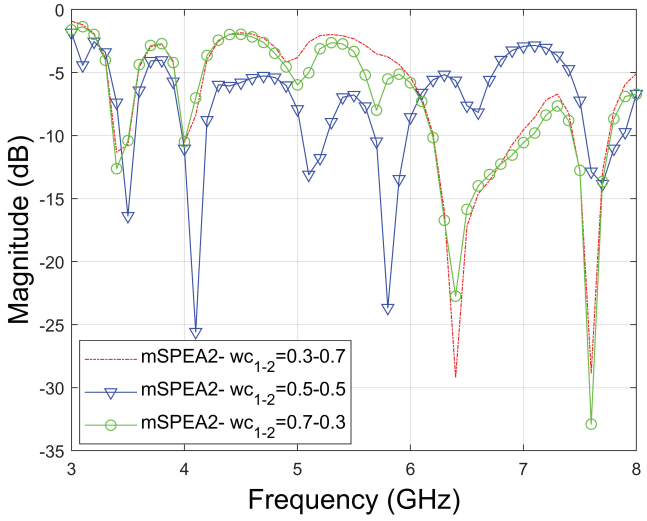


Fig. 5. Return loss of the fractal antenna obtained using MATLAB.

variable weighting coefficients using algorithms supported by different algorithm parameters and variant models. CST simulations were performed in accordance with the antenna design parameters, using the default CST parameters. In this context, mesh resolution, solution type, boundary conditions, and solution settings were determined using CST's automatic/default values. The material properties used were defined as PEC (conductor) and FR4 (substrate). This approach ensures the repeatability of the simulation and the reliability of the results. For the purpose of validating the findings that were discovered, a fractal patch was constructed and then simulated in the 3D electromagnetic simulation tool within the CST Microwave studio environment for a particular model. Here, the patch size in the results found with mSPEA2 in Table 5 is used. The 3D model of the antenna designed using the CST program is shown in Fig. 6. In Fig. 7, Table 5 the return loss (S_{11}) variation obtained with the CST program is given as a typical magnitude-frequency. Additionally, routing at 3.5 GHz, 4.1 GHz, 5.1 GHz, 5.8 GHz, and 7.7 GHz is shown in Figs 7 and 8(a-e), respectively. The resulting graphs appear to confirm the MATLAB results.

5. Discussion

In optimization problems based on Pareto optimality, the algorithms used often offer multiple feasible solutions. While this provides flexibility, it makes the determination of the final solution dependent on user preferences. An incorrect definition of the objective and cost functions can lead to the selection of undesirable solutions from the Pareto perspective. The population-based structure of the algorithm

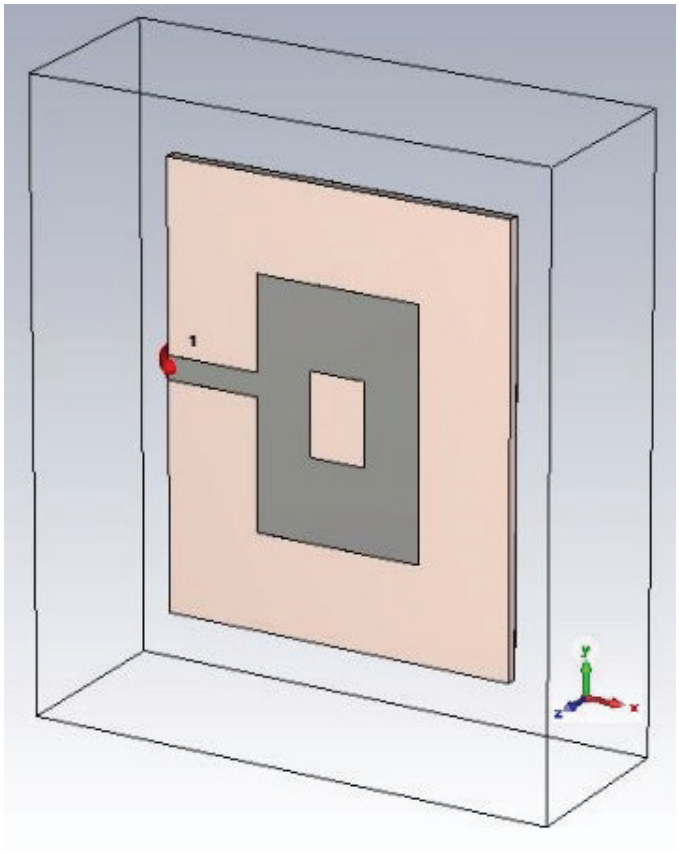


Fig. 6. 3D view of the fractal antenna designed with CST.

directly affects convergence behavior and optimization efficiency. Choosing a small population size can lead to premature convergence, while choosing a large one can increase computational costs and decrease efficiency. Therefore, appropriate population selection is critically important. Combining a dual objective function under a single cost function can limit the algorithm's search capability and reduce the variety of solutions. This can prevent the entire Pareto perspective from being represented and negatively impact the quality of convergence. Furthermore, minor changes to the antenna model can expand the solution space, increasing both the convergence speed and the quality of the solution. In conclusion, while this study includes innovations, it is open to improvement in terms of algorithm convergence, solution variety, and optimization efficiency.

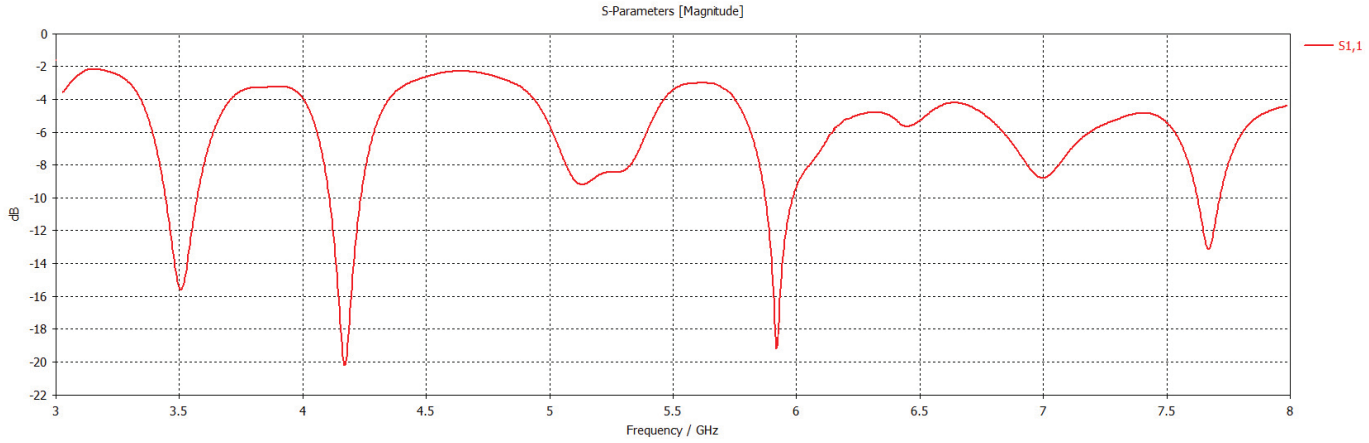


Fig. 7. Return loss simulation result of fractal antenna designed with CST using mSPEA2 design parameters.

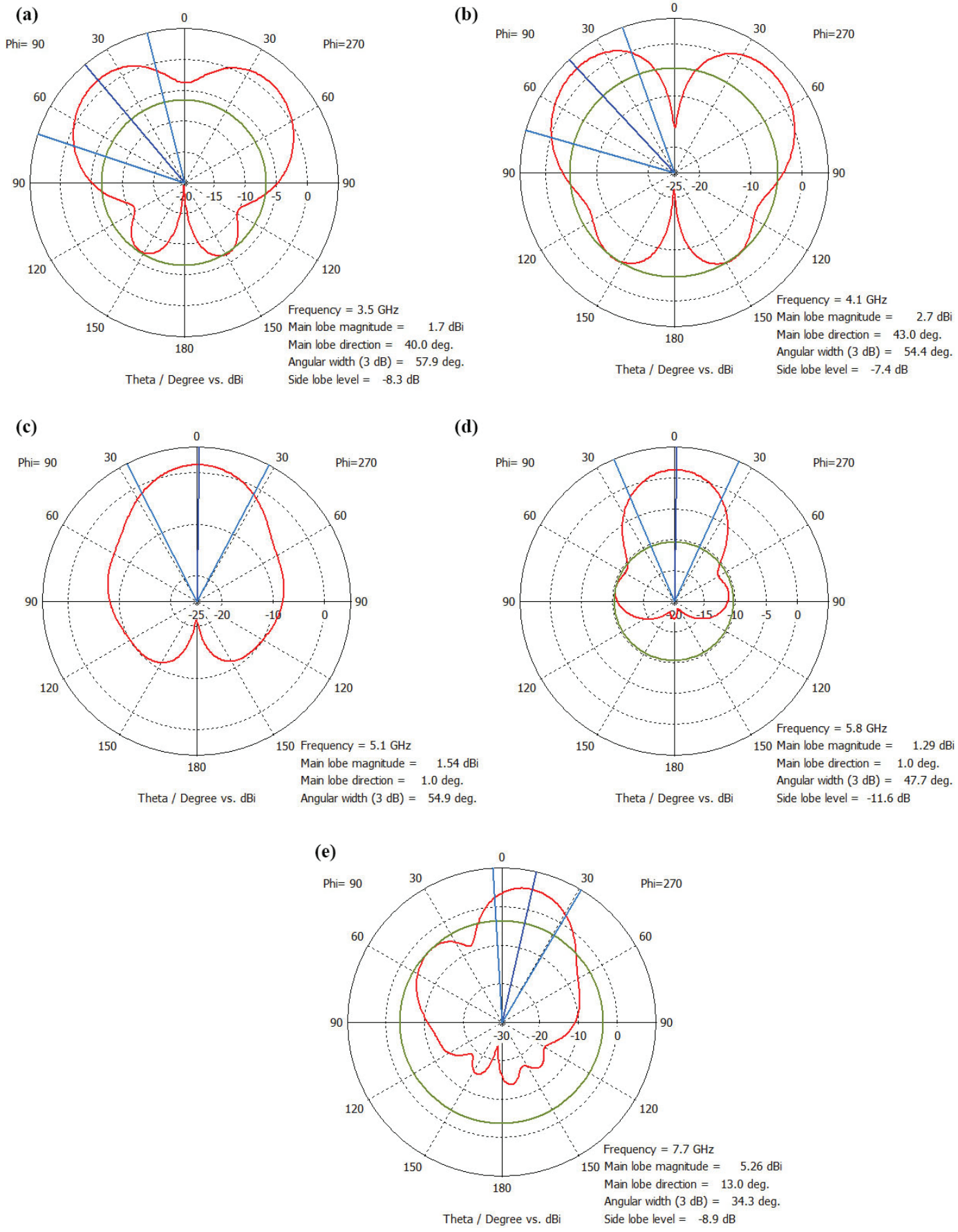


Fig. 8. Directivity of antenna for the mSPEA2 using CST (a) 3.5GHz, (b) 4.1GHz, (c) 5.1GHz, (d) 5.8GHz, (e) 7.7GHz.

6. Conclusions

In this study, Pareto optimization-based strength Pareto evolutionary algorithm methodology is discussed in detail, and variant models are revealed by making innovations in the algorithm of the existing model. The existing algorithm and these models are presented as a multidimensional and multi-objective optimization problem by using the existing algorithms and variants with 5 resonance frequencies (3.5 GHz, 4.1 GHz, 5.1 GHz, 5.8 GHz, and 7.7GHz). A carpet fractal antenna design with 6 variable design parameters has been studied. The first and second iterations of the SCFPA were used in the study. The proposed model is suitable for the increasing use of wireless networks and mobile communication systems. It also supports WLAN, Wi-Max, Wi-Fi, Bluetooth, etc. It also covers wireless applications. Since population diversity in evolutionary algorithms is very important for the efficiency of Pareto-optimal results and the algorithm used is population-based, special attention has been given to population parameter selection in order to both use resources efficiently and not get stuck with a local optimum. The pair of objective functions used was adapted from the nonlinear mathematical model and supported by different weighting coefficients. Since the optimization algorithms are based on randomly started conditions and processes, the algorithm code is run 10 times, as in other optimization problems. Since the SPEA2 model preferred in the study is a very rooted algorithm, it was supported by SPEA2 variant models. The performance of the developed variant algorithms is compared to that of the existing algorithm, which helps to reaffirm the success of the variant algorithms. The utilization of variant models was essential in the discovery of additional performance outcomes. The most successful results were obtained with weight coefficients (wc_{1-2}=0.5-0.5) using the variant mSPEA2. In the last part of the study, all these optimization processes are compared as a whole, and the design of the most successful result is presented as a 3D EM simulation.

The study shows that researchers can easily grasp the SPEA2 algorithm and can be quickly adapted for any multi-objective optimization problem. Although SPEA2 is the continuation of SPEA, it is seen that it has not completed its development yet. Better results can be obtained by suggesting new derivative versions in future studies. Studies on different types of objective functions can be done to improve performance. In addition, Artificial neural network (ANN) can be added to the problem to reduce the optimization time, which has the highest importance. Finally, variant algorithms can be easily used for any microwave optimization problem by arranging the objective functions.

In future studies, the following steps are suggested to increase the accuracy and applicability of the proposed optimization results: Performing performance verification through measurements by producing a real antenna prototype; testing the designed antenna in real wireless communication systems and evaluating its performance on a system-by-system basis; and conducting a more comprehensive evaluation by detailing the integration of the antenna into real communication systems and application scenarios.

CRediT authorship contribution statement

All the authors contributed to conceptualization, experimentation, analysis, validation, formal analysis, resources, writing-original draft preparation, writing-review and editing, funding acquisition, review, editing, software, writing, review, and final editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of generative AI and AI-assisted technologies in the writing process

The authors confirm that there was no use of artificial intelligence (AI)-assisted technology for assisting in the writing or editing of the manuscript, and no images were manipulated using AI.

Acknowledgement

This work is supported by the TUBITAK for founding our research under project number 225E050.

References

Agarwal, M., Srinivasan, R., Mishra, A., 2001. Study on flocculation efficiency of okra gum in sewage waste water. Macromol Mater Eng 286, 560-563, https://doi.org/10.1002/1439-2054(20010901)286:93.0.CO;2-B.

Agarwal, M., Rajani, S., Mishra, A., Rai, J.S.P., 2003. Utilization of okra gum for treatment of tannery effluent. Int J Polym Mater 52, 1049-1057. https://doi.org/10.1080/714975900

Agarwal, M., Kumar, R., Saxena, L.K., Tripathi, A., Singh, R., 2025. Comparative analysis of the treatment potential of cationic starch and its polyacrylamide blend on tannery and sewage effluents. AJC 37, 2532-2544. https://doi.org/10.14233/ajchem.2025.34437

Aggarwal, A., Kartikeyan, M.V., 2010. Design of sierpinski carpet antenna using two different feeding mechanisms for WLAN applications. 2010 35th International Conference on Infrared, Millimeter, and Terahertz Waves (IRMMW-THz 2010) Rome, Italy, 1-3. https://doi.org/10.1109/icimw.2010.5612727

Akyol, S., Alatas, B., 2017. Plant intelligence based metaheuristic optimization algorithms. Artif Intell Rev 47, 417-462. https://doi.org/10.1007/s10462-016-9486-6

Alatas, B., Bingol, H., 2020. Comparative assessment of light-based intelligent search and optimization algorithms. L&E, 51-59. https://doi.org/10.33383/2019-029

Almadi, A.I.M., Al Mamlook, R.E., Almarhabi, Y., Ullah, I., Jamal, A., Bandara, N., 2022. A Fuzzy-logic approach based on driver decision-making behavior modeling and simulation. Sustainability 14, 8874. https://doi.org/10.3390/su14148874

Azaro, R., Debiasi, L., Zeni, E., Benedetti, M., Rocca, P., Massa, A., 2009. A hybrid prefractal three-band antenna for multistandard mobile wireless applications. Antennas Wirel Propag Lett 8, 905-908. https://doi.org/10.1109/lawp.2009.2028627

Babu, B.S.S.V.R., Gullala, D., Srinivas, M.S.S.S., Gupta, A.K., Duvvarapu, P., Chimirala, C., Jaka, S., 2024. Design and analysis of square fractal shaped planar antenna. 2024 IEEE Wireless Antenna and Microwave Symposium (WAMS) Visakhapatnam, India, 1-4. https://doi.org/10.1109/wams59642.2024.10527930

Balanis, C., 2016. Antenna theory: Analysis and design, 4th ed. New York: John Wiley Sons.

Banerjee, J., Ghatak, R., Karmakar, A., 2018. A compact planar UWB MIMO diversity antenna with Hilbert fractal neutralization line for isolation improvement and dual band notch characteristics. 2018 Emerging Trends in Electronic Devices and Computational Techniques (EDCT) Kolkata, India, 1-6. https://doi.org/10.1109/edct.2018.8405070

Chaouche, Y.B., Nedil, M., Hammache, B., Belazzoug, M., 2019. Design of modified sierpinski gasket fractal antenna for Tri-band applications. 2019 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting Atlanta, GA, USA, 889-890. https://doi.org/10.1109/apusncursinrsm.2019.8889038

Chapra, S., Canale, R., 2001. Numerical methods for engineers, 8th ed. Boston: McGraw Hill.

Chen, K., Bao, Z., Liu, Y., Li, Y., 2026. Accurate and efficient pattern synthesis using generative adversarial network for series-fed microstrip antenna array with new elements. IEEE Trans. Antennas Propagat 74, 2833-2838. https://doi.org/10.1109/tap.2025.3641763

Çolak, B., Belen, M.A., Kiani, F., Tari, O., Mahouti, P., Akgol, O., 2026. A microstrip monopole antenna design for 5G Sub-6 GHz applications using deep learning. IET Communications 20. https://doi.org/10.1049/cmu2.70127

Dahbi, E.K., Elhamadi, T., Amar Touhami, N., 2022. Optimization of the SIW cavity-backed slots antenna for X-band applications using the particle swarm optimization algorithm. J Electromagnetic Waves Applications 36, 928-939. https://doi.org/10.1080/09205071.2021.1996278

Das, J., Sanjeeva Reddy, B.R., Reddy, T.A., Vishal, Y., Srilatha, Y., Reddy, N.U., 2023. RBF-neural network based implementation of sierpinski carpet shaped fractal antenna. 2023 IEEE 3rd International Conference on Technology, Engineering, Management for Societal Impact Using Marketing, Entrepreneurship and Talent (TEMSMET) Mysuru, India, 1-5. https://doi.org/10.1109/temsmet56707.2023.10150209

Duc Hieu, N., Chien, D.N., Khac Kiem,N., 2010. Flexible PSO-based optimization of millimeter-wave triple-band antennas by the use of fractal configuration. 2010 International Conference on Advanced Technologies for Communications (ATC 2010) Ho Chi Minh City, Vietnam, 336-340. https://doi.org/10.1109/atc.2010.5672676

Elahi, M., Koziel, S., Leifsson, L., 2026. Isolation and circular polarization diversity in 2×1 MIMO microstrip patch array using closed metallic loop. IEEE Access 14, 21072-21085. https://doi.org/10.1109/access.2026.3661731

Ge, Z., Zhou, X., Zhou, T., Sheng, H., Ren, J., Wang, Y., 2024. Design of a fractal-based ultra-wideband antenna. 2024 IEEE 10th International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications (MAPE) Guangzhou, China, 1-4. https://doi.org/10.1109/mape62875.2024.10813877

Ghatak, R., Mishra, R.K., Poddar, D.R., 2008. Perturbed sierpinski carpet antenna with CPW feed for IEEE 802.11 a/b WLAN application. Antennas Wirel Propag Lett 7, 742-744. https://doi.org/10.1109/lawp.2008.2004815

Gocen, C., Akdag, I., Mahouti, T., Belen, M.A., Palandöken, M., Mahouti, P., 2024. Knowledge-based methodology of CPW-Fed open stub loaded c-shaped microstrip antenna by surrogate-based modeling. Int J RF Microw Computer-Aided Eng 2024. https://doi.org/10.1155/2024/6247693

Gupta, K.C., 2000. Microstrip antenna design handbook, 1st ed. London: Artech house,

Hall, P., 2009. Keeping a low profile: The history of microstrip antennas. Propagation Conference (LAPC) Loughborough, UK, vi-vi. https://doi.org/10.1109/lapc.2009.5352362
Hillier, F., Lieberman, G., 2005. Introduction to operations research, 11th ed. New York: McGraw Hill.
Islam, Z.U., Rahman, M., Muhammad, N., Ahmed, Z., Lodhi, F.K., Haneef, M., 2016. Dual band second order modified Sierpinski monopole reduced size fractal antenna. 2016 International Conference on Emerging Technologies (ICET) Islamabad, Pakistan, 1-5. https://doi.org/10.1109/icet.2016.7813244
K, M., Santhanam, S., 2024. Antenna for C band applications using a modified sierpinski carpet fractal patch. 2024 Second International Conference on Microwave, Antenna and Communication (MAC) Dehradun, India, 1-6. https://doi.org/10.1109/mac61551.2024.10837060
Kattimani, B., R.Patil, R., 2023. Gain and bandwidth enhancement of sierpinski carpet based fractal patch antenna using stacked configuration. 2023 2nd International Conference for Innovation in Technology (INOCON) Bangalore, India, 1-5. https://doi.org/10.1109/inocon57975.2023.10101022
Kirubavathy, P.S., Ramprakash, K., 2017. Design of sierpinski fractal antenna for wideband applications. 2017 4th International Conference on Innovations in Information, Embedded and Communication Systems (ICIIECS) Coimbatore, 1-4. https://doi.org/10.1109/iciiecs.2017.8276063
Koniammal, R., Ramesh, G.P., Naidu, G.G., 2023. "Design a multiband frequency of partial defected ground for hexagonal fractal antenna," In 2023 international conference on data science and network security (ICDSNS), IEEE, 2023, 1-7. https://doi.org/10.1109/ICDSNS58469.2023.10244918.
Koziel, S., Saraereh, O., Jayasinghe, J.W., Uduwawala, D., 2017. Local optimization of a Sierpinski carpet fractal antenna. 2017 IEEE International Conference on Industrial and Information Systems (ICIIS) Peradeniya, 1-5. https://doi.org/10.1109/iciinfs.2017.8300415
Kumari, S., Hossain, A., 2024. Design of compact quad band circular slot microstrip patch antenna for S band, C band, X band, and Ku band satellite communication. 2024 IEEE Silchar Subsection Conference (SILCON) Agartala, India 1-4. https://doi.org/10.1109/silcon63976.2024.10910465
Lamsalli, M., El Hamichi, A., Boussouis, M., Amar Touhami, N., Elhamadi, T., 2016. Genetic algorithm optimization for microstrip patch antenna miniaturization. Pier Letters 60, 113-120. https://doi.org/10.2528/pierl16041907
Liu, H., Liu, Y., Wang, L., Han, J., Fang, X., Huang, J., Liu, J., 2026. A high-gain microstrip antenna based on microcoaxial technology for sub-terahertz applications. IEEE Trans. Antennas Propagat 74, 1440-1449. https://doi.org/10.1109/tap.2025.3639497
Lizzi, L., Massa, A., 2011. Dual-band printed fractal monopole antenna for LTE applications. Antennas Wirel Propag Lett 10, 760-763. https://doi.org/10.1109/lawp.2011.2163051
Long, S.A., O'Connor, E.M., 2016. The early history of the dielectric resonator antenna. 2016 IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting Fajardo, PR, USA, 679-680. https://doi.org/10.1109/aps.2016.7696048
Mahatmanto, P.A., Irawadi, D., Gunawan, H., Jatmiko, N.W., Sirin, D.N.S., Supriyono, Prabowo, Y., Akbar, A., Apriono, C., 2023. Design of C-band microstrip fractal antenna combined with pyramidal horn. 2023 International Conference on Radar, Antenna, Microwave, Electronics, and Telecommunications (ICRAMET) Bandung, Indonesia, 375-379. https://doi.org/10.1109/icramet60171.2023.10366770.
Mahouti, T., Yilmazer, H., Belen, M.A., 2026. AI-assisted 3D-printed meander-line antenna for non-invasive bone-tumor sensing. Sens Imaging 27. https://doi.org/10.1007/s11220-026-00750-6
Mainkar, S.D., 2023. Performance comparison of triangular KOCH curve and SIERPINSKI gasket fractal antenna designs for wireless applications. 2023 4th International Conference on Electronics and Sustainable Communication Systems (ICESC) Coimbatore, India, 567-571. https://doi.org/10.1109/icesc57686.2023.10193539
Milovanovic, B., Stankovic, Z., Milijic, M., 2006. Hybrid empirical-neural model of microwave slotted patch antennas. 2006 8th Seminar on Neural Network Applications in Electrical Engineering belgrade, Serbia, 181-184. https://doi.org/10.1109/neurel.2006.341207
Mishra, A., Rajani, S., Agarwal, M., Dubey, R., 2002. "P.psyllium-g-polyacrylamide: Synthesis and characterization," Polymer Bulletin 48, 439-444, Jul. 2002, https://doi.org/1010.1007/s00289-002-0056-5.
Mishra, A., Agarwal, M., Yadav, A., 2003. Fenugreek mucilage as a flocculating agent for sewage treatment. Colloid Polym Sci 281, 164-167. https://doi.org/10.1007/s00396-002-0765-1
Mishra, A., Yadav, A., Agarwal, M., Bajpai, M., 2004. Fenugreek mucilage for solid removal from tannery effluent. Reactive and Functional Polymers 59, 99-104. https://doi.org/10.1016/j.reactfunctpolym.2003.08.008
Mishra, A., Yadav, A., Agarwal, M., Rajani, S., 2004. Polyacrylonitrile-grafted Plantago psyllium mucilage for the removal of suspended and dissolved solids from tannery effluent. Colloid & Polymer Sci 282, 300-303. https://doi.org/10.1007/s00396-003-0895-0
Muwal, P.K., Saini, A.K., Choudhary, K., Beniwal, V., Pani, B., Sirohi, S., Mittal, A., Kumar, R., Singh, R., 2025. Development of cyclic and acyclic fluorescent chemosensors for selective detection of ions and biomolecules in water. ChemistrySelect 10. https://doi.org/10.1002/slct.202404394
Niyaz, Ö, Tüylü, T., Mahouti, P., Erkmen, B., Türker Tokan, N., 2024. Detection and classification of human respiration under building debris model using VHF/UHF waves. Neural Comput & Applic 36, 14709-14725. https://doi.org/10.1007/s00521-024-09848-y
Palandöken, M., Belen, A., Tari, O., Mahouti, P., Mahouti, T., Belen, M.A., 2024. Computationally efficient design optimization of multiband antenna using deep learning–based surrogate models. Int J RF Microw Computer-Aided Eng 2024. https://doi.org/10.1155/mmce/5442768
Piltan, O.C., Kizilay, A., Belen, M.A., Mahouti, P., 2024. Data driven surrogate modeling of horn antennas for optimal determination of radiation pattern and size using deep learning. Micro Optical Tech Lett. 66 ,e33702. https://doi.org/10.1002/mop.33702
Qiong, W., Ling, S., Hui, H.Y., 2024. "A design of broadband dual-polarization antenna based on fractal geometry," in 2024 14th International symposium on antennas, propagation and EM theory (ISAPE), IEEE, 1-3. https://doi.org/10.1109/ISAPE62431.2024.10840439.
Ramkumar, V., Kalaivani, S., Vanaja, S., Sheela, J.J.J., Deepika, Y., 2021. Design of a compact dual band microstrip reconfigurable antenna For WLAN/WiMAX application. 2021 Fifth International Conference on I-SMAC (IoT in Social, Mobile, Analytics and Cloud) (I-SMAC) Palladam, India, 1-6. https://doi.org/10.1109/i-smac52330.2021.9640714
Rao, N., Dinesh Kumar V., 2014. Modified sierpinski fractal patch antenna. 2014 loughborough Antennas & Propagation Conference (LAPC) Loughborough, Leicestershire, United Kingdom, 570-573. https://doi.org/10.1109/lapc.2014.6996454
Reeves, C.R., Rowe, J.E., 2002. Genetic algorithms—principles and perspectives, 20. Boston, MA: Springer US, https://doi.org/10.1007/b101880.
Romeu, J., Soler, J., 2001. Generalized sierpinski fractal multiband antenna. IEEE Trans Antennas Propagat 49, 1237-1239. https://doi.org/10.1109/8.943320
Roy, B., Bhattacharya, A., Chowdhury, S.K., Bhattacharjee, A.K., 2016. Optimization of resonant frequency of a sierpinski triangular CMPA using genetic algorithm. 2016 International Conference on Microelectronics, Computing and Communications (MicroCom) Durgapur, India, 1-5. https://doi.org/10.1109/microcom.2016.7522570
Sankhe, A.R., Khot, U.P., 2018. Optimization of iteration order and iteration factor in sierpinski carpet fractal patch antenna. 2018 International Conference on Smart City and Emerging Technology (ICSCET) Mumbai, 1-5. https://doi.org/10.1109/icscet.2018.8537388
Shankar, R., Singh, R., Kociok-Köhn, G., Molloy, K.C., 2014. Studies on the coordination behavior of methanesulfonate in Zinc(II) based two and three dimensional supramolecular assemblies. Proc Natl Acad Sci India Sect A Phys Sci 84, 157-163. https://doi.org/10.1007/s40010-013-0121-5
Shankar, R., Singh, R., Mendiratta, S., Jassal, A.K., Kociok-Köhn, G., Molloy, K.C., 2017. Synthesis and structural studies of three-dimensional supramolecular motifs derived from neutral and cationic zinc alkanesulfonates. Eur J Inorg Chem 2017, 2081-2087. https://doi.org/10.1002/ejic.201601320
Shankar, R., Mendiratta, S., Jakhar, E., Singh, R., Jassal, A.K., Kociok-Köhn, G., 2018. A rational synthesis of ladder-like motif in zinc-methylphosphonate from a preformed coordination assembly. Inorganica Chimica Acta 482, 681-686. https://doi.org/10.1016/j.ica.2018.07.016
Shankar, R., Singh, R., Kociok-Köhn, G., Frontera, A., 2020. A combined experimental and computational study of a supramolecular assembly based on cationic zinc(II)-ethanesulfonate. J Mol Struct 1202, 127206. https://doi.org/10.1016/j.molstruc.2019.127206
Shrestha, S., Jo Han, S., Noh, S.K., Kim, S., Kim, H.B., Choi, D.Y., 2013. Design of modified Sierpinski fractal based miniaturized patch antenna. 2013 International Conference on Information Networking (ICOIN) Bangkok, 274-279. https://doi.org/10.1109/icoin.2013.6496389
Singh, A.K., Kabeer, R.A., Ali, Z., Gurjar, D., 2013. Performance analysis of compact Koch fractal antennas at varying iterations. 2013 Students Conference on Engineering and Systems (SCES) Allahabad,1-5. https://doi.org/10.1109/sces.2013.6547565
Singh, R., Singh, K., Pandey, S.K., 2018. A computational scrutiny on the stability, structure, and electronic features of alkanesulfonate based zincate salts with varying countercations. ChemistrySelect 3, 13048-13056. https://doi.org/10.1002/slct.201803175
Singh, R., Kociok-Köhn, G., Singh, K., Pandey, S.K., Singh, L., 2019. Influence of ligand coordination, solvent, and non-covalent interaction on the structural outcomes in coordination polymers with direct Cd(II)-alkanesulfonate bonds: A combined experimental and computational study. J Solid State Chem 280, 120992. https://doi.org/10.1016/j.jssc.2019.120992
Singh, R., Kociok-Köhn, G., Kaur Jassal, A., Singh, L., 2020. Three-dimensional hydrogen-bonded magnesium(II) supramolecular motifs based on in situ generated alkanesulfonate (Me/Et/PrSO3−) ligands: A combined experimental and computational study. Polyhedron 175, 114200. https://doi.org/10.1016/j.poly.2019.114200
Sinha, A., Samyuktha, K.R., Mandal, J., Panakkal, V.P., 2024. Low-cost ultra-wideband fractal antenna array for L-band beam squint applications. 2024 IEEE Microwaves, Antennas, and Propagation Conference (MAPCON) Hyderabad, India, 1-4. https://doi.org/10.1109/mapcon61407.2024.10923052
Uduwawala, D., Jayasinghe, J., Narampanawe, N., 2019. Genetically designed high gain sierpinski carpet fractal antenna. 2019 IEEE 14th Conference on Industrial and Information Systems (ICIIS) Kandy, Sri Lanka, 11-14. https://doi.org/10.1109/iciis47346.2019.9063309
Uluslu, A., 2023. Fitting nonlinear mathematical models to the cost function of the quadrafilar helix antenna optimization problem. Analog Integr Circ Sig Process 115, 307-318. https://doi.org/10.1007/s10470-023-02174-8
Xu, Y.C., Xi, R.H., Tong, M.S., 2024. A fractal prediction method for multiband patch antenna based on quasi-mandelbrot fractal geometry. 2024 IEEE international conference on microwaves, communications, antennas, biomedical engineering,

and electronic systems (COMCAS) Tel Aviv, Israel, 1-3. https://doi.org/10.1109/comcas58210.2024.10666247

Yadav, S., Chourasiya, S., Bhagwat, T., Chaitanya, G., 2018. Modified sierpinski carpet semi-fractal CPW feed antenna for wireless applications. 2018 8th International Conference on Communication Systems and Network Technologies (CSNT) Bhopal, India,2, 6-9. https://doi.org/10.1109/csnt.2018.8820283

Yang, H., Gemeay, E., Alawad, M.A., Alkaoud, M., Lee, S., Elsaid, S.A., 2025. Utilizing convolutional neural network and gray wolf optimization for image super-resolution. J King Saud Univ Sci 37, 1622024. https://doi.org/10.25259/jksus_162_2024

Yi, L.Y., Wang, A.Z., Zhang, L., Fang, S., Hui, C.T., 2023. A circularly polarized microstrip phased array antenna for 5G communication. 2023 16th UK-Europe-China Workshop on Millimetre Waves and Terahertz Technologies (UCMMT) Guangzhou, China 1-3. https://doi.org/10.1109/ucmmt58116.2023.10310309

Yu, Z.W., Wang, G.M., Gao, X.J., Lu, K., 2010. A novel small-size single patch microstrip antenna based on Koch and sierpinski fractal-shapes. Pier Letters 17, 95-103. https://doi.org/10.2528/pierl10062803

Yurt, R., Torpi, H., Kizilay, A., Koziel, S., Mahouti, P., 2024. Variable data structures and customized deep learning surrogates for computationally efficient and reliable characterization of buried objects. Sci Rep 14, 14898. https://doi.org/10.1038/s41598-024-65996-0

Zheng, X., Chen, X., 2023. Microstrip antenna based on fractal structure. 2023 21st international conference on optical communications and networks (ICOCN) Qufu, China, 1-3. https://doi.org/10.1109/icocn59242.2023.10236397

Zitzler, E., Laumanns, M., Thiele, L., 2001. “SPEA2: Improving the strength pareto evolutionary algorithm”. https://sop.tik.ee.ethz.ch/publicationListFiles/zlt2001a.pdf